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Branching fractions of $B_{(c)}$ decays involving J/ψ and X(3872) *

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Abstract: We study two-body $B_{(c)} \to M_c(\pi, K)$ and semileptonic $B_c \to M_c l^- \bar{\nu}_l$ decays with $M_c = (J/\psi, X_c^0)$, where $X_c^0 \equiv X^0(3872)$ is regarded as the tetraquark state $c\bar{c}u\bar{u}(d\bar{d})$. With the decay constant $f_{X_c^0} = (234 \pm 52)$ MeV determined from the data, we predict that $\mathcal{B}(B^- \to X_c^0 \pi^-) = (11.5 \pm 5.7) \times 10^{-6}$, $\mathcal{B}(\bar{B}^0 \to X_c^0 \bar{K}^0) = (2.1 \pm 1.0) \times 10^{-4}$, and $\mathcal{B}(\bar{B}_s^0 \to X_c^0 \bar{K}^0) = (11.4 \pm 5.6) \times 10^{-6}$. With the form factors in QCD models, we calculate that $\mathcal{B}(B_c^- \to X_c^0 \pi^-, X_c^0 K^-) = (6.0 \pm 2.6) \times 10^{-5}$ and $(4.7 \pm 2.0) \times 10^{-6}$, and $\mathcal{B}(B_c^- \to J/\psi \mu^- \bar{\nu}_\mu, X_c^0 \mu^- \bar{\nu}_\mu) = (2.3 \pm 0.6) \times 10^{-2}$ and $(1.35 \pm 0.18) \times 10^{-3}$, respectively, and extract the ratio of the fragmentation fractions to be $f_c/f_u = (6.4 \pm 1.9) \times 10^{-3}$.

Keywords: B decays, B_c decays, J/ψ, X(3872)

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1 Introduction

Through the $b \to c\bar{c}d(s)$ transition at quark level, B decays are able to produce $c\bar{c}$ bound states like J/ψ; particularly, the hidden charm tetraquarks to consist of $c\bar{c}q\bar{q}'$, such as $X^0(3872)$, Y(4140), and $Z_c^+(4430)$, known as the XYZ states [1]. For example, we have [2, 3]

$$\mathcal{B}(B^- \to J/\psi K^-) = (1.026 \pm 0.031) \times 10^{-3}, \\ \mathcal{B}(B^- \to X_c^0 K^-) = (2.3 \pm 0.9) \times 10^{-4}, \quad (1)$$

where $X_c^0 \equiv X^0(3872)$ is composed of $c\bar{c}u\bar{u}(d\bar{d})$, measured to have the quantum numbers $J^{PC} = 1^{++}$. On the other hand, the B_c^- decays from the $b \to c\bar{u}d(s)$ transition can also be a relevant production mechanism for the $c\bar{c}$ and $c\bar{c}q\bar{q}'$ bound states. However, the current measurements have been done only for the ratios, given by [4, 5]

$$\mathcal{R}_{c/u} \equiv \frac{f_c \mathcal{B}(B_c^- \to J/\psi \pi^-)}{f_u \mathcal{B}(B^- \to J/\psi K^-)} = (0.68 \pm 0.12)\%, \\ \mathcal{R}_{K/\pi} \equiv \frac{\mathcal{B}(B_c^- \to J/\psi K^-)}{\mathcal{B}(B_c^- \to J/\psi \pi^-)} = 0.069 \pm 0.020, \\ \mathcal{R}_{\pi/\mu\bar{\nu}_\mu} \equiv \frac{\mathcal{B}(B_c^- \to J/\psi \pi^-)}{\mathcal{B}(B_c^- \to J/\psi \mu^- \bar{\nu}_\mu)} = (4.69 \pm 0.54)\%, \quad (2)$$

where $f_{c,u}$ are the fragmentation fractions defined by $f_i \equiv \mathcal{B}(b \to B_i)$. In addition, none of the XYZ states have been observed in the B_c decays yet.

From Figs. 1(a) and 1(d), the $B \to M_c M$ decays proceed by the $B \to M$ transition, which is followed by the recoiled $M_c = (J/\psi, X_c^0)$ with $J^{PC} = (1^{--,++})$, respectively, presented as the matrix elements of $\langle M_c | \bar{c}\gamma_\mu(1-\gamma_5)c|0\rangle$. Unlike J/ψ, which is a genuine $c\bar{c}$ bound state, while the matrix element for the tetraquark production is in fact not computable, X_c^0 is often taken as a charmonium state in the QCD models [6–8]. In this study, we will extract $\langle X_c^0 | \bar{c}\gamma_\mu(1-\gamma_5)c|0\rangle$ from the data of $\mathcal{B}(B^- \to X_c^0 K^-)$ in Eq. (1) to examine the decays of $B^- \to X_c^0(\pi^-, K^-)$, $\bar{B}^0 \to X_c^0(\pi^-, K^-)$, and $\bar{B}_s^0 \to X_c^0 K^-$, of which the extraction allows X_c^0 to be the tetraquark state. On the other hand, to calculate the $B_c^- \to (J/\psi, X_c^0)M$ decays in Figs. 1(b) and 1(e) and the semileptonic $B_c^- \to (J/\psi, X_c^0) l \bar{\nu}_l$ decays in Figs. 1(c) and 1(f), we use the $B_c \to M_c$ transition matrix elements from the QCD calculations.

2 Formalism

In terms of the effective Hamiltonians at quark level for the $b \to c\bar{c}q$, $b \to c\bar{u}q$, and $b \to cl\bar{\nu}_l$ transitions in Fig. 1, the amplitudes of the $B_c^- \to M_c M$, $B \to M_c M$, and $B_c^- \to M_c l^- \bar{\nu}_l$ decays can be factorized as [9, 10]

$$\mathcal{A}(B_c^- \to M_c M) \\ = i\frac{G_F}{\sqrt{2}} V_{cb} V_{uq}^* a_1 f_M \langle M_c | \bar{c} \not{q} (1-\gamma_5) b | B_c^- \rangle,$$

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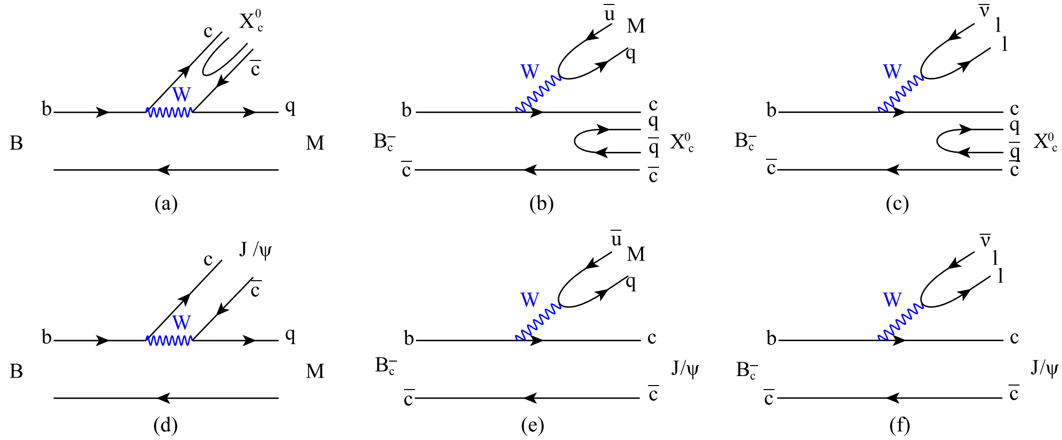

Fig. 1. Diagrams for the B and B_c decays with formation of the $c\bar{c}$ pair, where (a), (b) and (c) correspond to the $B\to X_c^0 M$, $B_c^-\to X_c^0 M$, and $B_c^-\to X_c^0 l\bar{\nu}_l$ decays, while (d), (e) and (f) the $B\to J/\psi M$, $B_c^-\to J/\psi M$, and $B_c^-\to J/\psi l\bar{\nu}_l$ decays, respectively.

$$\begin{aligned}
&\mathcal{A}(B\to M_c M)\\
&=\frac{G_F}{\sqrt{2}}V_{cb}V^*_{cq}a_2 m_{M_c} f_{M_c}\langle M|\bar{q}\not{\varepsilon}(1-\gamma_5)b|B\rangle\,,\\
&\mathcal{A}(B_c^-\to M_c l^-\bar{\nu}_l)\\
&=\frac{G_F V_{cb}}{\sqrt{2}}\langle M_c|\bar{c}\gamma_\mu(1-\gamma_5)b|B_c^-\rangle\bar{l}\gamma^\mu(1-\gamma_5)\nu_l, \qquad (3)
\end{aligned}$$

respectively, where $\not{q}=q^\mu\gamma_\mu$, $\not{\varepsilon}=\varepsilon^{\mu*}\gamma_\mu$, q = d(s) for $M=\pi^-(K^-)$, $M_c=(J/\psi, X_c^0)$, $l=(e^-,\mu^-,\tau^-)$, G_F is the Fermi constant, and V_{ij} are the CKM matrix elements. In the factorization approach, $a_{1(2)}\equiv c^{\rm eff}_{1(2)}+c^{\rm eff}_{2(1)}/N_c$ is composed of the effective Wilson coefficients in Ref. [9], with $(c_1^{\rm eff}, c_2^{\rm eff})=(1.168,-0.365)$, where N_c is the color number. In Eq. (3), the decay constant, four-momentum vector, and four polarization $(f_{M_{(c)}}, q^\mu, \varepsilon^{\mu*})$ are defined by

$$\begin{aligned}
\langle M|\bar{q}\gamma_\mu\gamma_5 u|0\rangle&=-if_M q^\mu\,,\\
\langle J/\psi|\bar{c}\gamma_\mu c|0\rangle&=m_{J/\psi}f_{J/\psi}\varepsilon^*_\mu\,,\\
\langle X_c^0|\bar{c}\gamma_\mu\gamma_5 c|0\rangle&=m_{X_c^0}f_{X_c^0}\varepsilon^*_\mu\,, \qquad (4)
\end{aligned}$$

while the matrix elements of the $B\to(M, J/\psi, X_c^0)$ transitions can be parametrized as [8]

$$\begin{aligned}
\langle M|\bar{q}\gamma^\mu b|B\rangle&=\left[(p_B+p_M)^\mu-\frac{m_B^2-m_M^2}{t}q^\mu\right]F_1^{BM}(t)\\
&\quad+\frac{m_B^2-m_M^2}{t}q^\mu F_0^{BM}(t)\,,\\
\langle J/\psi|\bar{c}\gamma_\mu b|B_c^-\rangle&=\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}p^\alpha_{B_c}p^\beta_{J/\psi}\frac{2V(t)}{m_{B_c}+m_{J/\psi}}\,,\\
\langle J/\psi|\bar{c}\gamma_\mu\gamma_5 b|B_c^-\rangle&=i\left[\varepsilon^*_\mu-\frac{\varepsilon^*\cdot q}{t}q_\mu\right](m_{B_c}+m_{J/\psi})A_1(t)\\
&\quad+i\frac{\varepsilon^*\cdot q}{t}q_\mu(2m_{J/\psi})A_0(t)\\
&\quad-i\left[(p_{B_c}+p_{J/\psi})_\mu-\frac{m_{B_c}^2-m_{J/\psi}^2}{t}q_\mu\right]\\
&\quad(\varepsilon^*\cdot q)\frac{A_2(t)}{m_B+m_{J/\psi}}\,,\\
\langle X_c^0|\bar{c}\gamma_\mu\gamma_5 b|B_c^-\rangle&=-\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}p^\alpha_{B_c}p^\beta_{X_c^0}\frac{2iA(t)}{m_{B_c}-m_{X_c^0}}\,,\\
\langle X_c^0|\bar{c}\gamma_\mu b|B_c^-\rangle&=-\left[\varepsilon^*_\mu-\frac{\varepsilon^*\cdot q}{t}q_\mu\right](m_{B_c}-m_{X_c^0})V_1(t)\\
&\quad-\frac{\varepsilon^*\cdot q}{t}q_\mu(2m_{X_c^0})V_0(t)\\
&\quad+\left[(p_{B_c}+p_{X_c^0})_\mu-\frac{m_{B_c}^2-m_{X_c^0}^2}{t}q_\mu\right]\\
&\quad(\varepsilon^*\cdot q)\frac{V_2(t)}{m_B-m_{X_c^0}}\,, \qquad (5)
\end{aligned}$$

respectively, where $q=p_B-p_{M_{(c)}}$, $t\equiv q^2$, and $(F_{1,2}, A_{(i)}, V_{(i)})$ with $i=0,1,2$ are the form factors.

3 Numerical results and discussions

In our numerical analysis, we use the Wolfenstein parameterization for the CKM matrix elements in Eq. (3), given by $V_{cb}=A\lambda^2$, $V_{ud}=V_{cs}=1-\lambda^2/2$, and $V_{us}=-V_{cd}=\lambda$, with [2]

$$(\lambda, A, \rho, \eta)=(0.225, 0.814, 0.120\pm0.022, 0.362\pm0.013). \qquad (6)$$

In the generalized version of the factorization [9], though $N_c=3$, it is allowed to float from 2 to ∞, which empirically estimates the uncertainty from the non-factorizable effects, such that one has $a_1=1.05^{+0.12}_{-0.06}$ [11] in $B_c^-\to M_c M$. Since a_2 in $B\to M_c M$ is sensitive to non-factorizable effects, it relies on the extraction from $B^-\to J/\psi K^-$ to give $a_2=0.268\pm0.004$ [12]. The decay constants and form factors adopted from Refs. [2, 13] and [8, 14] are as follows:

$$(f_\pi, f_K, f_{J/\psi}) = (130.4\pm0.2, 156.2\pm0.7, 418\pm9)\,\text{MeV},$$
$$(F_1^{B\pi}(0), F_1^{BK}(0), F_1^{BsK}(0)) = (0.29, 0.36, 0.31), \quad (7)$$

where the form factors correspond to the reduced matrix elements derived from Eqs. (3) and (5), given by

$$\langle M|\bar{q}\not{\varepsilon} b|B\rangle = \varepsilon\cdot(p_B+p_M)F_1^{BM}. \quad (8)$$

The momentum dependence for $F_1^{BM}(q^2)$ from Ref. [14] is taken as

$$F_1^{BM}(t) = \frac{F_1^{BM}(0)}{\left(1-\frac{t}{M_V^2}\right)\left(1-\frac{\sigma_{11}t}{M_V^2}+\frac{\sigma_{12}t^2}{M_V^4}\right)}, \quad (9)$$

with $\sigma_{11} = (0.48, 0.43, 0.63)$, $\sigma_{12} = (0, 0, 0.33)$ and $M_V = (5.32, 5.42, 5.32)$ GeV for $B\to\pi$, $B\to K$ and $\bar{B}^0_s\to K$, respectively. With $\mathcal{B}(B^-\to X_c^0K^-)/\mathcal{B}(B^-\to J/\psi K^-) = 0.22\pm0.09$ from Eq. (1), we obtain $f_{X_c^0} = (234\pm52)$ MeV, which is lower than $f_{X_c^0} = (335, 329^{+111}_{-95})$ MeV [7, 8] from perturbative and light-front QCD models, respectively. The momentum dependences for the $B_c\to M_c$ transition form factors are given by [15]

$$f(t) = f(0)\exp(\sigma_1 t/m^2_{B_c} + \sigma_2 t^2/m^4_{B_c}), \quad (10)$$

where the values of $f(0) = (V_{(i)}(0), A_{(i)}(0))$ and $\sigma_{1,2}$ in Table 1 are from Refs. [8] and [15], respectively. Our results for the branching ratios of $B_c^-\to J/\psi(\pi^-, K^-, l^-\bar{\nu}_l)$ are shown in Table 2.

Table 1. The $B_c\to(J/\psi, X_c^0)$ form factors at $t=0$ and $\sigma_{1,2}$ for the momentum dependences in Eq. (10).

$B_c\to(J/\psi, X_c^0)$	$f(0)$ [8]	σ_1	σ_2	[15]
(V, A)	$(0.87\pm0.02, 0.36\pm0.04)$	2.46	0.56	
(A_0, V_0)	$(0.57\pm0.02, 0.18\pm0.03)$	2.39	0.50	
(A_1, V_1)	$(0.55\pm0.03, 1.15\pm0.07)$	1.73	0.33	
(A_2, V_2)	$(0.51\pm0.04, 0.13\pm0.02)$	2.22	0.45	

From Table 2, we see that our numerical values of $\mathcal{B}(B_c^-\to J/\psi\pi^-)$ and $\mathcal{B}(B_c^-\to J/\psi K^-)$ are about a factor 2 smaller than those in Ref. [8], where the calculations were done only by the leading-order contributions in the $1/m_{B_c}$ expansion[1]. We also note that, by carefully computing the non-factorizable effects, it is given that $\mathcal{B}(B_c^-\to J/\psi\pi^-) = (29.1^{+1.5+4.0}_{-4.2-2.7})\times10^{-4}$ and $\mathcal{B}(B_c^-\to J/\psi K^-) = (22^{+1+3}_{-3-2})\times10^{-5}$ [16], which are around 2 times as large as our results. From the table, we get that $\mathcal{B}(B_c^-\to J/\psi\pi^-)/\mathcal{B}(B_c^-\to J/\psi K^-) = 0.078\pm0.027$, which agrees with $\mathcal{R}_{K/\pi}$ in Eq. (2), demonstrating the validity of the factorization approach. By taking $\mathcal{B}(B_c^-\to J/\psi\pi^-)$ as the theoretical input in Eq. (2), we find that

$$f_c/f_u = (6.4\pm1.9)\times10^{-3}, \quad (11)$$

which can be useful to determine the experimental data, such as those in Eq. (2).

Table 2. The branching ratios of the $B_c\to J/\psi(M, l\bar{\nu}_l)$ decays, where the first (second) errors of our results are from the form factors (a_1).

decay modes	our results	QCD models
$B_c^-\to J/\psi\pi^-$	$(10.9\pm0.8^{+2.6}_{-1.2})\times10^{-4}$	$(20^{+8+0+0}_{-7-1-0})\times10^{-4}$ [8]
$B_c^-\to J/\psi K^-$	$(8.8\pm0.6^{+2.1}_{-1.0})\times10^{-5}$	$(16^{+6+0+0}_{-6-1-0})\times10^{-5}$ [8]
$B_c^-\to J/\psi e^-\bar{\nu}_e$	$(1.94\pm0.20)\times10^{-2}$	$(1.49^{+0.01+0.15+0.23}_{-0.03-0.14-0.23})\times10^{-2}$ [15]
$B_c^-\to J/\psi \mu^-\bar{\nu}_\mu$	$(1.94\pm0.20)\times10^{-2}$	$(1.49^{+0.01+0.15+0.23}_{-0.03-0.14-0.23})\times10^{-2}$ [15]
$B_c^-\to J/\psi \tau^-\bar{\nu}_\tau$	$(4.47\pm0.48)\times10^{-3}$	$(3.70^{+0.02+0.42+0.56}_{-0.05-0.38-0.56})\times10^{-3}$ [15]

For the $B\to X_c^0(\pi, K)$ decays, the results are given in Table 3. While $f_{X_c^0} = (234\pm52)$ MeV leads to $\mathcal{B}(B^-\to X_c^0K^-) = (2.3^{+1.1}_{-0.9}\pm0.1)\times10^{-4}$ in accordance with the data, we predict that $\mathcal{B}(B^-\to X_c^0\pi^-) = (11.5\pm5.7)\times10^{-6}$, $\mathcal{B}(\bar{B}^0\to X_c^0\bar{K}^0) = (2.1\pm1.0)\times10^{-4}$,

Table 3. The branching ratios for the $B_{(c)}\to X_c^0$ M and $B_c\to X_c^0 l\bar{\nu}_l$ decays. For our results, the first errors come from $(f_{X_c^0}, f(0))$, and the second ones from (a_1, a_2).

decay modes	our results	QCD models
$B^-\to X_c^0\pi^-$	$(11.5^{+5.7}_{-4.5}\pm0.3)\times10^{-6}$	—
$B^-\to X_c^0K^-$	$(2.3^{+1.1}_{-0.9}\pm0.1)\times10^{-4}$	$(7.88^{+4.87}_{-3.76})\times10^{-4}$ [7]
$\bar{B}^0\to X_c^0\pi^0$	$(5.3^{+2.6}_{-2.1}\pm0.2)\times10^{-6}$	—
$\bar{B}^0\to X_c^0\bar{K}^0$	$(2.1^{+1.0}_{-0.8}\pm0.1)\times10^{-4}$	—
$\bar{B}^0_s\to X_c^0\bar{K}^0$	$(11.4^{+5.6}_{-4.5}\pm0.3)\times10^{-6}$	—
$B_c^-\to X_c^0\pi^-$	$(6.0^{+2.2+1.4}_{-1.8-0.7})\times10^{-5}$	$(1.7^{+0.7+0.1+0.4}_{-0.6-0.2-0.4})\times10^{-4}$ [8]
$B_c^-\to X_c^0K^-$	$(4.7^{+1.7+1.1}_{-1.4-0.5})\times10^{-6}$	$(1.3^{+0.5+0.1+0.3}_{-0.5-0.2-0.3})\times10^{-5}$ [8]
$B_c^-\to X_c^0e^-\bar{\nu}_e$	$(1.35\pm0.18)\times10^{-3}$	$(6.7^{+0.9+0.0+0.1+0.5+2.3+0.7}_{-0.5-0.0-0.0-0.5-2.6-0.7})\times10^{-3}$ [19]
$B_c^-\to X_c^0\mu^-\bar{\nu}_\mu$	$(1.35\pm0.18)\times10^{-3}$	—
$B_c^-\to X_c^0\tau^-\bar{\nu}_\tau$	$(6.5\pm0.9)\times10^{-5}$	$(3.2^{+0.5+0.0+0.0+0.2+1.1+0.4}_{-0.2-0.2-0.0-0.2-1.3-0.3})\times10^{-4}$ [19]

1) We thank the authors in Ref. [8] for the useful communication.

and $\mathcal{B}(\bar{B}^0_s \to X^0_c\bar{K}^0) = (11.4 \pm 5.6) \times 10^{-6}$, which are accessible to the experiments at the LHCb. Besides, our results of $\mathcal{B}(\bar{B}^0_s \to X^0_c\bar{K}^0) \simeq \mathcal{B}(B^- \to X^0_c\pi^-)$ and $\mathcal{B}(\bar{B}^0 \to X^0_c\pi^0) \simeq \mathcal{B}(B^- \to X^0_c\pi^-)/2$ in Table 3 are also supported by the $SU(3)$ and isospin symmetries, respectively. With the form factors adopted from Ref. [8], we calculate that $\mathcal{B}(B^-_c \to X^0_c\pi^-) = (6.0 \pm 2.6) \times 10^{-5}$ and $\mathcal{B}(B^-_c \to X^0_c K^-) = (4.7 \pm 2.0) \times 10^{-6}$, which are 2–3 times smaller than the results from the same reference. The differences are again reconciled after keeping the next-leading order contributions in the $1/m_{B_c}$ expansion.

For the semileptonic $B^-_c \to M_c l^-\bar{\nu}_l$ decays, $\mathcal{B}(B^-_c \to J/\psi e\bar{\nu}_e) = \mathcal{B}(B^-_c \to J/\psi\mu\bar{\nu}_\mu) = (1.94 \pm 0.20) \times 10^{-2}$ is due to the both negligible electron and muon masses, of which the numerical value is close to those from Refs. [15, 17] but $2-3$ times smaller than those in Ref. [18], which calls for future experimental examination. Note that by taking $\mathcal{B}(B^-_c \to J/\psi\pi^-)$ as the theoretical input in Eq. (2), we derive that

$$\mathcal{B}(B^-_c \to J/\psi\mu^-\bar{\nu}_\mu) = (2.3 \pm 0.6) \times 10^{-2}, \qquad (12)$$

which agrees with the above theoretical prediction. For the τ mode, which suppresses the phase space due to the heavy m_τ, we obtain $\mathcal{B}(B^-_c \to J/\psi\tau^-\bar{\nu}_\tau) = (4.47 \pm 0.48) \times 10^{-3}$. The ratio of $\mathcal{B}(B^-_c \to X^0_c e^-\bar{\nu}_e)/\mathcal{B}(B^-_c \to X^0_c\tau^-\bar{\nu}_\tau) \simeq 1/20$ is close to that in Ref. [19], but $\mathcal{B}(B^-_c \to X^0_c e^-\bar{\nu}_e) = (1.35 \pm 0.18) \times 10^{-3}$ is apparently 4-5 times smaller than that in Ref. [19], though with uncertainties the two results overlap with each other. With the spectra of $B^-_c \to (J/\psi, X^0_c)l^-\bar{\nu}_l$ in Fig. 2, our results can be compared to the recent studies on the semileptonic B_c cases in Refs. [20, 21] for the XYZ states.

4 Conclusions

In sum, we have studied the $B_{(c)} \to M_c(\pi, K)$ and $B_c \to M_c l^-\bar{\nu}_l$ decays with $M_c = J/\psi$ and $X^0_c \equiv X^0(3872)$. We have presented that $\mathcal{B}(B^- \to X^0_c\pi^-, X^0_c K^-) = (11.5 \pm 5.7) \times 10^{-6}$ and $(2.3 \pm 1.1) \times 10^{-4}$, and $\mathcal{B}(B^-_c \to X^0_c\pi^-, X^0_c K^-) = (6.0 \pm 2.6) \times 10^{-5}$ and $(4.7 \pm 2.0) \times 10^{-6}$. With $\mathcal{B}(B^-_c \to J/\psi\pi^-) = (10.9 \pm 2.6) \times 10^{-4}$ as the theoretical input, the extractions from the data have shown that $f_c/f_u = (6.4 \pm 1.9) \times 10^{-3}$ and $\mathcal{B}(B^-_c \to J/\psi\mu^-\bar{\nu}_\mu) = (2.3 \pm 0.6) \times 10^{-2}$. We have estimated $\mathcal{B}(B^-_c \to X^0_c l^-\bar{\nu}_l)$ with $l = (e^-, \mu^-, \tau^-)$ to be $(1.35 \pm 0.18) \times 10^{-3}$, $(1.35 \pm 0.18) \times 10^{-3}$, and $(6.5 \pm 0.9) \times 10^{-5}$, respectively.

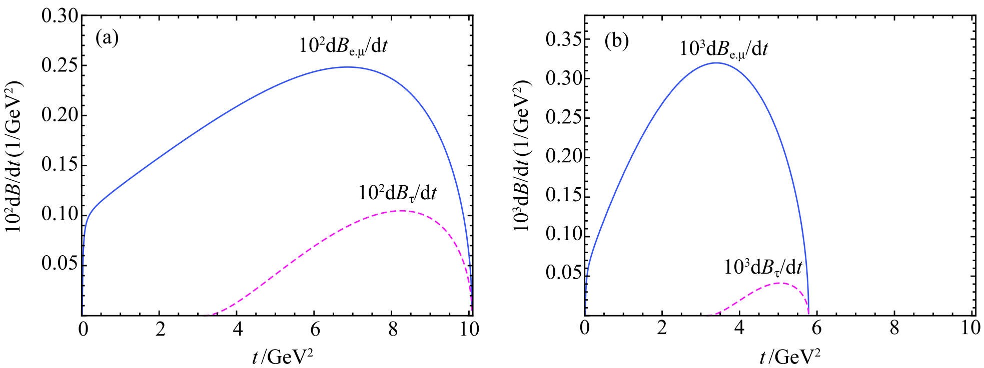

Fig. 2. (color online) The spectra of the semileptonic (a) $B^-_c \to J/\psi l^-\bar{\nu}_l$ and (b) $B^-_c \to X^0_c l^-\bar{\nu}_l$ decays, where the solid and dotted lines correspond to $l = (e, \mu)$ and $l = \tau$, respectively.

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