On the production of hidden-flavored hadronic states at high energy[*]

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Abstract: I discuss the production mechanism of hidden-flavored hadrons at high energy. Using e^+e^- collisions and light-meson pair production in high energy exclusive processes, I demonstrate that hidden quark pairs do not necessarily participate in short-distance hard scattering. Implications are then explored in a few examples. Finally, I discuss the production mechanism of X(3872) in hadron collisions, where some misunderstandings have arisen in the literature.

1   Introduction

In the past few decades, hadron physics, in particular the study of exotic hadrons, has been the subject of extensive theoretical and experimental interest. For recent reviews, see Refs. [1–3]. On the experimental side, quite a number of candidates for exotic hadrons have been observed. Those include not only mesonic states like the charged $Z_c(3900)^\pm$ [4, 5] as four-quark candidates but also baryonic states $P_c(4380)$ and $P_c(4450)$ [6], which are likely pentaquarks. These exciting experimental observations have stirred much theoretical interest [1–3]. On the one hand, QCD allows scenarios other than the usual scheme of a meson made of quark-anti-quark and a baryon as a system of three quarks. On the other hand, due to the nonperturbative nature, it is very difficult to have a model-independent analysis of the internal structure of candidates for these hadrons.

The production of hadrons at high energy typically involves several different scales. It is widely believed that the hard momentum exchange is calculable using perturbation theory. However, recently there has been a debate on how to understand the production of the X(3872) [7–9]. The fact that X(3872) can be copiously produced in hadron collisions has led to suspicion of the molecular assignment of the X(3872). Reference [7] has used Monte Carlo simulation and calculated the production rates of $D\bar{D}^*$. Using a momentum cutoff set by the binding energy, the authors have found the simulated cross section is smaller than the data by orders of magnitude. Such a choice of the momentum cutoff is questioned in Ref. [8], while a comment on this questioning appeared in Ref. [9]. In addition, the authors of Ref. [9] have used the production data of a deuteron (a loosely bound state) in a previous study [10]. They argued that if X(3872) is also a molecule, one expects the production of X(3872) and deuterons to have similar behaviors. Comparing the data on production of deuterons and X(3872) at hadron colliders, they have found differences and thus argued that "The results suggest a different production mechanism for the X(3872), making questionable any loosely bound molecule interpretation" [10].

In this work, I will show that the production mechanism of the X(3872) is not properly understood in Refs. [9, 10]. To do so, I will first use standard processes, the $e^+e^- \to \rho^0\pi^0$ and B, D decays, and show that high energy production does not always reveal the hadron's low-energy structure. This will induce differences in the production of light nuclei like the deuteron and X(3872). Finally, I will briefly comment on the production mechanism of X(3872) at hadron level and propose a new conjectured mechanism.

The rest of this paper is organized as follows. In Section 2, I point out that naive applications can lead to wrong interpretations of the hadron structure [11–15]. I give the correct way from the viewpoint of effective field theory [16, 17]. In semileptonic B and D decays into a pair of light mesons, I will show that hidden quarks do

not participate in the scattering either. This is similar to the $B_c \to X(3872)$ transition [18]. Section 3 concentrates on the X(3872). A short summary is presented in the last section.

2 Hard exclusive reactions: e^+e^- collision and B, D decays

In a high energy reaction, if factorization exists, short-distance and long-distance degrees of freedom decouple. For an exclusive process like $e^+e^- \to \rho^0\pi^0$, the constituent scaling rule is a consequence of perturbative QCD analysis, which has been derived in a number of classic papers [19–21]. In the following, I will first present a more convenient derivation using a modern effective field theory approach, soft-collinear effective theory (SCET). Using SCET, I explicitly demonstrate that the naive constituent scaling rules must be remedied in the case of hidden-flavored hadrons.

2.1 SCET

SCET can be used to study processes involving light hadrons at high energy [22]. Instead of directly studying the s dependence, we introduce a dimensionless parameter $\lambda = \Lambda/\sqrt{s}$ and and count the power dependence on λ

$$\frac{d\sigma}{dt} \sim \frac{1}{s^2}\left(\frac{\Lambda}{\sqrt{s}}\right)^n. \tag{1}$$

The scale Λ is a low-energy scale and may be taken as Λ_{QCD} in the case of a light quark, or $m_{c/b}$ in the case of a charm/bottom quark if involved.

At high energy, the energetic quarks or gluons are jet-like (collinear) with the typical momentum

$$p = (p^+, p^-, p_\perp) \propto \left(\sqrt{s}, \frac{\Lambda^2}{\sqrt{s}}, \Lambda\right). \tag{2}$$

For an energetic quark, it is convenient to split the quark field ψ into two components:

$$\psi = \xi + \eta, \quad \xi = \frac{\not{n}\not{\bar{n}}}{4}\psi, \quad \eta = \frac{\not{\bar{n}}\not{n}}{4}\psi,$$

where n and $\bar{n}$ are two light-like vectors: $n^2 = \bar{n}^2 = 0$. The quark field scaling can be obtained by considering the two-point correlator:

$$\langle 0|T[\psi(x)\overline{\psi}(y)]|0\rangle = \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(x-y)} \frac{i(\not{p}+m)}{p^2-m^2+i\epsilon}. \tag{3}$$

This gives

$$\xi \propto \lambda, \eta \propto \lambda^2. \tag{4}$$

For a collinear photon/gluon field, one has the propaga-

tor in the general R_ξ gauge as:

$$\langle 0|T[A_\mu(x)A_\nu(y)]|0\rangle = \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(x-y)} \frac{-i}{p^2-m^2+i\epsilon}$$
$$\times \left[g_{\mu\nu}-(1-\xi)\frac{p_\mu p_\nu}{p^2}\right]. \tag{5}$$

Then one finds the scaling:

$$n_+ A \propto 1, \quad A_\perp \propto \lambda, \quad n_- A \propto \lambda^2. \tag{6}$$

In the following, we will not encounter a soft gluon/photon.

A mesonic/leptonic state scales as $|M\rangle \propto \lambda^{-1}$, which can be easily derived from the normalization of states:

$$\langle M(p)|M(p')\rangle = (2\pi)^3 2E_p \delta^3(\vec{p}-\vec{p}'). \tag{7}$$

For a lepton, scalings of state and field will cancel, and thus one only needs to consider the final hadron.

2.2 $e^+e^- \to \rho^+\pi^-$ and $e^+e^- \to \rho^0\pi^0$

At high energy with $\sqrt{s} \gg \Lambda_{\text{QCD}}$, exclusive processes are calculable in perturbation theory. When factorization holds, one may separate the interactions according to the scales involved using the operator product expansion. The interactions above the factorization scale can be integrated out, which results in an effective field theory. We show the matching for the $e^+e^- \to \rho^+\pi^-$ in Fig. 1. The photon propagator, quark propagator and gluon propagator are highly off-shell, and thus these propagators can be shrunk to the same space-coordinate. Then in low energy effective field theory the cross section is factorized as:

$$\mathcal{M}(e^+e^- \to \rho^+\pi^-) = C \otimes \langle\pi^+|\bar{\xi}_n \frac{\not{\bar{n}}}{2}\xi_n|0\rangle \otimes \langle\rho^+|\bar{\eta}_{\bar{n}}\gamma_\perp\xi_{\bar{n}}|0\rangle. \tag{8}$$

Since the ρ^+ is transversely polarized, the small component η contributes. n and $\bar{n}$ are two unit light-like vectors with $n^2 = \bar{n}^2 = 0$, $n\bar{n} = 1$. Here $\otimes$ denotes a convolution in the space coordinate, and C is an $\mathcal{O}(1)$ coefficient. Using the building blocks given in the last subsection, we have the power counting:

$$\mathcal{M}(e^+e^- \to \rho^+\pi^-) \propto \lambda^3. \tag{9}$$

The cross section scales as:

$$\sigma(e^+e^- \to \rho^+\pi^-) \propto \frac{1}{s}\lambda^6 \propto \frac{\Lambda^6}{s^4}. \tag{10}$$

This result is consistent with the perturbative QCD calculations [23, 24], and validated by experimental data [25–29]. The above result is also consistent with the classical constituent scaling rule [19–21]:

$$\sigma(e^+e^- \to \rho^+\pi^-) \propto \frac{1}{s^{n_t-3}} \times \frac{1}{s}, \tag{11}$$

where n_t denotes the total number of constituents in the process. Since ρ and π contains two quarks, we have

$n_t=1+1+2+2=6$. The last factor $1/s$ arises from helicity suppression.
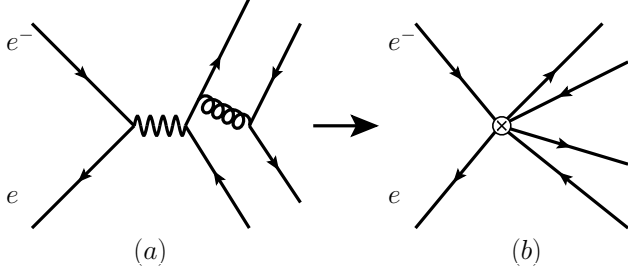

Fig. 1. Leading power Feynman diagram for the photon contribution to $e^+e^- \to \rho^+\pi^-$ in the full theory (a) and effective field theory (b).

Now we consider the $e^+e^- \to \rho^0\pi^0$. A vector meson such as ρ^0 can be produced by a photon field $\langle\rho^0|A_\perp^\mu|0\rangle$. Then the decay amplitude has the power scaling

$$\mathcal{M} \propto \langle\pi|\bar{\xi}_n \bar{n}\!\!\!/ \xi_n|0\rangle \times \langle\rho^0|A_\perp^\mu|0\rangle \propto \lambda, \quad (12)$$

which leads to the cross section

$$\sigma(e^+e^- \to \rho^0\pi^0) \propto \frac{1}{s}\lambda^2 \propto \frac{\Lambda^2}{s^2}. \quad (13)$$

This result contradicts the naive constituent scaling rule given in Eq. (11).

A few remarks are in order.

1) It is necessary to stress that the photon contribution is suppressed by the fine structure constant α_{em} and is less important at low energy. At very high energy the photon contribution is at leading power [16, 23]. It has also been shown that this mechanism will lead to important consequences in electroweak penguin-dominated B decays [30, 31].

2) To understand the above behaviors, one can count the valence degrees of freedom of the neutral vector meson as $n_i=1$, which amounts to counting the number of lines (a photon in this case, as shown in Fig. 2) attached to the effective vertex shown in Fig. 2.
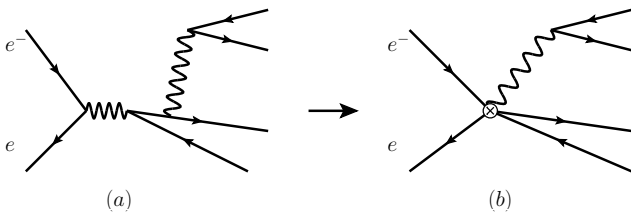

Fig. 2. Leading power Feynman diagram for the photon contribution to $e^+e^- \to \rho^0\pi^0$ in the full theory (a) and effective field theory (b). Unlike the $e^+e^- \to \rho^+\pi^-$ case, the two quarks in ρ^0 do not participate in the hard-scattering, and thus the leading power amplitude is not sensitive to the two-quark nature of ρ^0.

3) From the viewpoint of effective field theory, the nonzero matrix element $\langle\rho^0|A_\perp^\mu|0\rangle$ uses Heisenberg operators. When converting to the interaction picture, one must include the interactions:

$$\langle\rho^0|A_\perp^\mu|0\rangle \equiv \langle T[\rho^0|A_\perp^\mu \times \exp\left[i\int d^4x \mathcal{L}(x)\right]|0\rangle, \quad (14)$$

with the standard QED interaction Lagrangian

$$\mathcal{L} = \bar{q}ee_q A\!\!\!/ q. \quad (15)$$

4) The scale dependence of parton distribution function (PDF) is encoded in the DGLAP evolution. For a flavor singlet, the quark PDF and gluon PDF mix with each other. From this viewpoint, our results would be similar: the photon field operator at a high scale evolves to the quark-anti-quark field operator at a low scale. These operators will mix in the scale evolution.

5) The above example indicates that not all constituents in the hadron participate in the hard scattering. Therefore, one cannot use the scaling behavior of the cross section to decipher the hadron's structure.

2.3 $e^+e^- \to Z_c^\pm\pi^\mp$, $e^+e^- \to D_{s0}(2317)^\pm D_s^{*\mp}$ and $e^+e^- \to \phi f_0(980)$

After the discussion with an ordinary hadron, I now propose a few processes to explore the $Z_c(3900)^\pm$ [4, 5], $D_{s0}(2317)$ [32], and $f_0(980)$.

The $Z_c(3900)^\pm$ decays into $J/\psi\pi^\pm$, and the lowest Fock state is expanded as four quarks [4, 5]. If the quarks are democratically distributed, this is identified as a tetraquark meson. It is also likely that the $Z_c(3900)^\pm$ is made of two mesons, i.e. that it is a hadron molecule. For $e^+e^- \to Z_c^\pm\pi^\mp$ production, typical Feynman diagrams are shown in Fig. 3. These two diagrams (panels (a, c)) will compete. At low energy, the two charm quarks will be produced first, and then the light quarks are generated. The perturbative suppression for the production of light quarks might not be severe. So it is likely that the panel (a) dominates near threshold. However, at very high energy, panel (a) is suppressed due to the hard gluons, and the leading power matrix element is given in panel (c) with the factorization formula:

$$\mathcal{M}(e^+e^- \to Z_c^\pm\pi^\mp)$$
$$= C \otimes \langle\pi^+|\bar{\xi}_{n,d}\frac{\bar{n}\!\!\!/}{2}\xi_{n,u}|0\rangle \otimes \langle Z_c^-|\bar{\xi}_{\bar{n},u}\frac{n\!\!\!/}{2}\xi_{\bar{n},d}|0\rangle. \quad (16)$$

Thus we can predict that the cross section scales as

$$\sigma(e^+e^- \to Z_c^\pm\pi^\mp) \propto \frac{1}{s}\lambda^2 \propto \frac{\Lambda^4}{s^3}. \quad (17)$$

1) The above matrix elements in Eq. (16) are written in the Heisenberg picture. When converting to the interaction picture, one has to include the interaction below
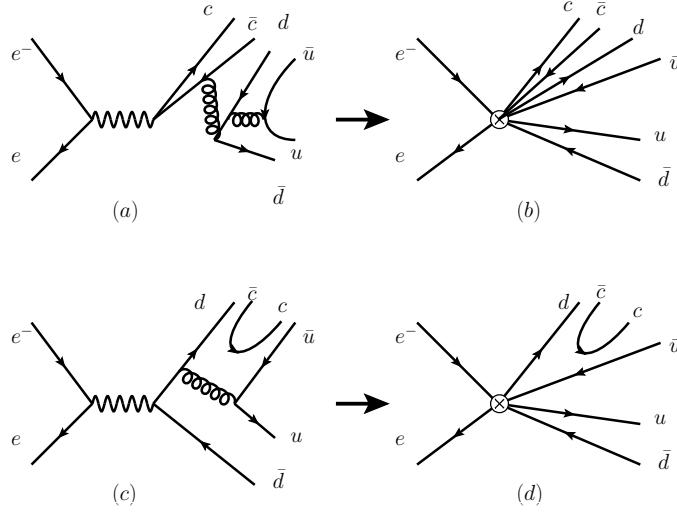
Fig. 3.   Feynman diagrams for $e^+e^- \to Z_c^\pm \pi^\mp$ in the full theory (a, c) and effective field theory (b, d).

the scale $\sqrt{s}$, and formally have

$$\langle Z_c^- | \bar{\xi}_{\bar{n},u} \frac{\slashed{n}}{2} \xi_{\bar{n},d} | 0 \rangle$$

$$= \langle Z_c^- | \bar{\xi}_{\bar{n},u} \frac{\slashed{n}}{2} \xi_{\bar{n},d} \times \exp[i \int d^4 x \mathcal{L}_{int}(x)] | 0 \rangle, \quad (18)$$

with the interaction Lagrangian:

$$\mathcal{L}_{int} = \bar{c} g \slashed{A} c + \bar{q} g \slashed{A} q. \quad (19)$$

Notice that unlike the ρ^0, one cannot handle this time-ordered product perturbatively. So Eq. (18) is a formal equation.

2) The production of $\bar{c}c$ in panel (c) is suppressed by $1/m_c^2$. Thus at low energy $\sqrt{s} \sim m_c$, both panel (a) and panel (c) contribute.

3) The reaction $e^+e^- \to D_{s0}(2317)^\pm D_s^{*\mp}$ as shown in Fig. 4 was observed for the first time with a data sample of 567 pb^{-1} collected with the BESIII detector operating at the BEPC-II collider at $\sqrt{s} = 4.6$ GeV [33]. The low collision energy does not guarantee the use of perturbation theory. However, we expect a study at Belle-II can uniquely test the same scaling behavior given in Eq. (17).
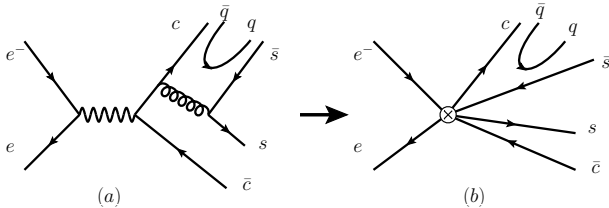

Fig. 4.   At high energy, Feynman diagrams for the $e^+e^- \to D_{s0}(2317)^+ D_s^{*-}$ in the full theory (a) and effective field theory (b).

4) $e^+e^- \to \phi f_0(980)$ can proceed similarly, with the Feynman diagrams given in Fig. 5. Experimentally, the

BESIII and Babar collaborations have used the initial state radiation and measured $e^+e^- \to \phi \pi^+ \pi^-$ [34, 35]. I suggest that our experimental colleagues study the collision energy dependence and validate the production mechanism in this work.
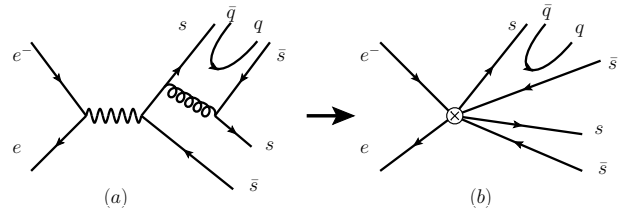

Fig. 5.   At high energy, Feynman diagrams for $e^+e^- \to \phi f_0(980)$ in the full theory (a) and effective field theory (b).

2.4   B, D decays into a light-meson pair and $\gamma\gamma$ fusion

Pairs of light pseudo-scalar mesons have a special relation with light scalar mesons, for instance $f_0(980)$ and $\kappa(800)$ [36]. Recently, there have been studies of semileptonic B,D decays into light meson pairs [37, 38]. A typical Feynman diagram for $D \to \pi^+ \pi^- e^+ \nu$ is given in Fig. 6, and for other channels, the Feynman diagrams are similar. In these decays, the leptonic sector can be factorized out and calculated perturbatively. The nonleptonic matrix element is then parameterized as:

$$\langle (\pi^+ \pi^-)_S (p_{\pi\pi}) | \bar{s} \gamma_\mu \gamma_5 c | D(p_D) \rangle$$

$$= -i \frac{1}{m_{\pi\pi}} \left\{ \left[P_\mu - \frac{m_D^2 - m_{\pi\pi}^2}{q^2} q_\mu \right] \mathcal{F}_1^{D \to \pi\pi}(m_{\pi\pi}^2, q^2) \right.$$

$$\left. + \frac{m_D^2 - m_{\pi\pi}^2}{q^2} q_\mu \mathcal{F}_0^{D \to \pi\pi}(m_{\pi\pi}^2, q^2) \right\}, \quad (20)$$

where we have only shown the S-wave $\pi\pi$ final state. $m_{\pi\pi}$ is the $\pi\pi$ invariant mass. This defines the S-wave generalized form factors $\mathcal{F}_i$ [39]. Here, $P=p_{\mathrm{D}}+p_{\pi\pi}$ and $q=p_{\mathrm{D}}-p_{\pi\pi}$.
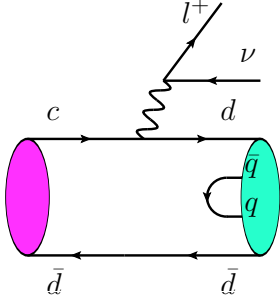

Fig. 6. (color online) A typical Feynman diagram for $\mathrm{D}\to\pi^+\pi^-\mathrm{e}^+\nu$. The leptonic sector can be calculated using perturbation theory, while the $\mathrm{D}\to\pi\pi$ transition is parameterized in terms of form factors.

The study of generalized form factors requires knowledge of the generalized light-cone distribution amplitude (LCDA). The leading twist LCDA of $\pi\pi$ systems is defined by two quark fields. The leading-power behavior in $1/s$ is then determined by the two quarks, irrespective of the structures of the and $\pi\pi$ systems [40]. Here notice the different dependence on the invariant mass $m_{\pi\pi}$ and the collision energy $\sqrt{s}$.

Using the light-cone sum rules, one can derive the factorization formula [39]:

$$\mathcal{F}_i(q^2,m_{\pi\pi}^2)=B_0 m_{\pi\pi}F_{\pi\pi}(m_{\pi\pi}^2)\overline{F}_i(m_{\pi\pi}^2,q^2), \quad (21)$$

where B_0 is the QCD condensate parameter and $F_{\pi\pi}(m_{\pi\pi}^2)$ is the $\pi\pi$ scalar form factor. The $\overline{F}_i(m_{\pi\pi}^2,q^2)$ is a function of the two-meson LCDA defined by

$$\langle\pi^+\pi^-|\bar{s}(x)(1,\gamma^\mu,\sigma^{\mu\nu})s|0\rangle. \quad (22)$$

This approach has recently been used to calculate heavy meson decays in Refs. [39, 41–48] and agreements with relevant data [37, 38] are found.
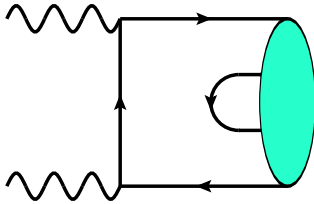

Fig. 7. (color online) Feynman diagrams for $\gamma\gamma\to\pi^+\pi^-/K^+K^-/\pi^0\pi^0$.

It should also be viable to study two-meson production in $\gamma\gamma$ processes at BESII [49] and Belle-II in future. Such processes are only sensitive to the leading twist generalized LCDA, and the Feynman diagram is given in Fig. 7. Actually, the Belle collaboration has published the first investigation of momentum dependence in the two-pion system [50]. The $\pi^+\pi^-$ system was studied for momentum transfers between $3\leq Q^2[\mathrm{GeV}^2]\leq30$. Again we should warn that the existing proposals to use this process and extract the structure of scalar mesons [51, 52] are problematic.

3   X(3872)

3.1   Differences between production of light nuclei and X(3872)

The authors of Ref. [9] have used production data for a deuteron (a loosely bound state) in a previous study [10]. They argued that if X(3872) is also a molecule, a similar scaling behavior in the production rate to that for a deuteron is expected. Using data for the production of deuterons and X(3872) at hadron colliders, they have found differences and thus argued that "The results suggest a different production mechanism for the X(3872), making questionable any loosely bound molecule interpretation" [10].

Actually, there are dramatic differences between the production of the deuteron and the X(3872). Unlike the deuteron, which contains 6 quarks, the X(3872) contains hidden flavors, and thus one cannot use the same power scalings for the two hadrons. Instead, the production rates of the X(3872) at high energy hadron colliders are determined by the quark-anti-quark field. The production rates do not scale according to its low energy structure, whether molecule or tetraquark.

The production of the X(3872) meson involves many length scales [53]. The creation of the $\bar{c}c$ pair with a small relative momentum requires a hard-scattering process at the scale m_c. This $\bar{c}c$ pair can be color singlet or color-octet. The evolution of the $\bar{c}c$ into a color-singlet hadron occurs over a softer scale $m_c v$ or $m_c v^2$. Then the evolution of the charmed mesons occurs over an even lower scale m_π. At last the binding of DD^* into the molecular state X occurs over a very long length scale. To calculate the production rates of X(3872) at high energy, one can employ the nonrelativistic QCD (NRQCD) approach:

$$\sigma(\mathrm{p\bar{p}}\to X) = \hat{\sigma}(\mathrm{p\bar{p}}\to\bar{c}c)\langle X|O_{c\bar{c}}|X\rangle, \quad (23)$$

with the $\hat{\sigma}(\mathrm{p\bar{p}}\to\bar{c}c)$ being the partonic cross section. The matrix elements $\langle X|O_{\bar{c}c}|X\rangle$ are low energy inputs, no matter whether the X(3872) is an ordinary charmonium, a hadron molecule, or a tetraquark. Based on the NRQCD framework, the next-to-leading order calculations [54, 55] are consistent with the ATLAS data for the production of X(3872) at $\sqrt{s}=8$TeV [56].

The production mechanism can be further tested in a number of processes. For the exclusive $\mathrm{e}^+\mathrm{e}^-\to\gamma X(3872)$

at high energy, the $\langle X|\bar{c}\Gamma c|0\rangle$ (Γ is a Dirac matrix) contributes, and the cross section should scale as $1/s^3$, derived from the two-quark structure in order to produce the X(3872). In $B_c \to X(3872)$ decays, the decay amplitude is irrespective of the emitted particles in B_c decays. Thus the ratios of branching fractions of semileptonic and nonleptonic decays, for instance $B_c \to X\ell\bar{\nu}$ and $B_c \to X\rho$, can be precisely predicted [18] and tested by data.

3.2   More on X(3872) production mechanism at hadron level

In Ref. [7], an inequality for the production rates of X(3872) has been derived:

$$\sigma(\bar{p}p \to X) \simeq \left| \int_{\mathcal{R}} d^3\boldsymbol{k} \langle X|D^0\bar{D}^{*0}(\boldsymbol{k})\rangle \langle D^0\bar{D}^{*0}(\boldsymbol{k})|\bar{p}p\rangle \right|^2$$
$$\leq \int_{\mathcal{R}} d^3\boldsymbol{k} |\Psi(\boldsymbol{k})|^2 \int_{\mathcal{R}} d^3\boldsymbol{k} \left| \langle D^0\bar{D}^{*0}(\boldsymbol{k})|\bar{p}p\rangle \right|^2$$
$$\leq \int_{\mathcal{R}} d^3\boldsymbol{k} \left| \langle D^0\bar{D}^{*0}(\boldsymbol{k})|\bar{p}p\rangle \right|^2. \qquad (24)$$

References [8, 9] have discussed the different choices of $\mathcal{R}$ in detail, resulting in dramatically different conclusions. In the following I will directly discuss the production mechanism.

The cross section for the inclusive process is defined as

$$\sigma(\bar{p}p \to X) = \int \frac{d^3 p_X}{(2\pi)^3 2E_X} \frac{d^3 p_{\text{anything}}}{(2\pi)^3 2E_{\text{anything}}} |M_{\bar{p}p \to X+\text{anything}}|^2$$
$$\times (2\pi)^4 \delta^4(\sqrt{s} - p_X - p_{\text{anything}}), \qquad (25)$$

where X denotes the X(3872) and the symbol "anything" denotes the remnant.

The amplitude is defined as (up to some kinematic factor)

$$M_{\bar{p}p \to X+\text{anything}} \sim \langle X+\text{anything}|T|\bar{p}p\rangle. \qquad (26)$$

If one wants to insert a unit operator, one cannot use

$$1 = \int d^3\boldsymbol{k} |D^0\bar{D}^{*0}(\boldsymbol{k})\rangle\langle D^0\bar{D}^{*0}(\boldsymbol{k})|, \qquad (27)$$

but instead one should use

$$1 = \int d^3\boldsymbol{k} d^3\boldsymbol{p}_{\text{anything}'} |D^0\bar{D}^{*0}(\boldsymbol{k})+\text{anything}'\rangle$$
$$\times \langle D^0\bar{D}^{*0}(\boldsymbol{k})+\text{anything}'|, \qquad (28)$$

where we have picked up the $D^0\bar{D}^{*0}(\boldsymbol{k})$. Inserting this unit operator into the matrix element, one has

$$M_{\bar{p}p \to X+\text{anything}} \sim \langle X+\text{anything}|T|\bar{p}p\rangle$$
$$= \int d^3\boldsymbol{k} d^3\boldsymbol{p}_{\text{anything}'} \langle X+\text{anything}|D^0\bar{D}^{*0}(\boldsymbol{k})+\text{anything}'\rangle$$
$$\times \langle D^0\bar{D}^{*0}(\boldsymbol{k})+\text{anything}'|T|\bar{p}p\rangle. \qquad (29)$$

If one assumes

$$\text{anything} = \text{anything}', \qquad (30)$$

one will recover the first line of Eq. (24). However this assumption is not trivial. An example is the production of J/ψ and other charmonium. If one assumes anything=anything', then the J/ψ is only produced by the $\bar{c}c$ state that has the same quantum numbers as J/ψ. But in the NRQCD approach, it is widely known that the J/ψ can also be produced by the color octet configurations, in which anything$\neq$anything'. Such contributions are found to be sizable.

The assumption anything=anything' for the production of X(3872) is equivalent to *local constituent-molecule duality*, namely, the production rate of the constituents in the phase space is equivalent to that of the molecule. This is similar to *local quark-hadron duality*, which often fails for very narrow resonances. To recover the quark-hadron duality, one should include final state interactions, which is equivalent to increasing the momentum cutoff [57, 58].

To calculate the production of ordinary heavy quarkonium, one often uses NRQCD, in which a hadron is nonperturbatively produced by quark fields. Similarly, if there is factorization, for hadronic molecules we may establish an approach in which the hadron molecule is produced by its constituents, and the low-energy matrix element has been estimated using an effective theory at hadron level. The cross section should have the conjectured form [59, 60]:

$$\sigma(p\bar{p} \to X) = \hat{\sigma}(p\bar{p} \to D\bar{D}^*)\langle X|O_{D\bar{D}^*}|X\rangle, \qquad (31)$$

where $\hat{\sigma}(p\bar{p} \to D\bar{D}^*)$ is the *partonic* cross section. The $\langle X|O_{D\bar{D}^*}|X\rangle$ is a low energy input and will only be determined in a nonperturbative way. This approach avoids the use of *local constituent-molecule duality*, and thus the results should be more reliable.

4   Conclusion

The study of production of exotic hadrons is an important facet of hadron physics. However, in the past decade, there have been great misunderstandings which have hindered us in correctly understanding the nature of hadron exotics. In this work, I have demonstrated that for a reaction involving hidden flavored hadrons, if there is factorization, short-distance and long-distance degrees of freedom may decouple from each other. Using $e^+e^- \to \rho^0\pi^0$ and a few other examples, I have shown that high energy production does not reveal the hadron's low-energy structure. This has important consequences in the study of the production of hadron exotics, in particular the X(3872). This should be a warning to our research community that the misuse of production data can lead to misleading results for the nature of exotic

hadrons.

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