Discovery potentials of doubly charmed baryons *

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Abstract: The existence of doubly heavy flavor baryons has not been well established experimentally so far. In this Letter we systematically investigate the weak decays of the doubly charmed baryons, Ξ_{cc}^{++} and Ξ_{cc}^{+}, which should be helpful for experimental searches for these particles. The long-distance contributions are first studied in the doubly heavy baryon decays, and found to be significantly enhanced. Comparing all the processes, $\Xi_{cc}^{++}\to\Lambda_c^+K^-\pi^+\pi^+$ and $\Xi_c^+\pi^+$ are the most favorable decay modes for experiments to search for doubly heavy baryons.

Keywords: doubly charmed baryon, weak decay, branching fraction, factorization, final-state interaction

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1 Introduction

Plenty of hadrons, including quite a few exotic candidates, have been discovered in experiments during the past few decades. Doubly and triply heavy flavor baryons, however, with two or three heavy (b or c) quarks, are so far still absent in hadron spectroscopy [1–4]. Searches for the doubly and triply heavy baryons will play a key role in completing hadron spectroscopy and shedding light on perturbative and non-perturbative QCD dynamics.

The only evidence was reported by the SELEX experiment for Ξ_{cc}^+ via the process $\Xi_{cc}^+\to\Lambda_c^+K^-\pi^+$ in 2002 [5], followed by $\Xi_{cc}^+\to pD^+K^-$ [6]. However, this has not been confirmed by any other experiments so far. The FOCUS experiment reported no signal right after SELEX's measurements [7]. The BaBar [8] and Belle [9, 10] experiments searched for $\Xi_{cc}^{+(+)}$ with the final states of $\Xi_c^0\pi^+(\pi^+)$ and $\Lambda_c^+K^-\pi^+(\pi^+)$, and did not find any evidence. The LHCb experiment performed a search using the $0.65\,\text{fb}^{-1}$ data sample in the discovery channel used by SELEX, $\Xi_{cc}^+\to\Lambda_c^+K^-\pi^+$, but no significant signal was observed [11]. Besides, the mass measured by SELEX, $m_{\Xi_{cc}^+}=3518.9\pm0.9$ MeV, is much lower than most theoretical predictions, for instance $m_{\Xi_{cc}}=$ 3.55–3.67 GeV predicted by lattice QCD [12–17]. These puzzles can only be solved by experimental measurements with high luminosities.

At the Large Hadron Collider (LHC), plenty of heavy quarks have been generated, and thereby abundant doubly heavy hadrons have been produced due to quark-hadron duality, such as $B_c^{\pm}$, which has been studied in great detail by LHCb. The cross sections of the hadronic production of Ξ_{cc} at the LHC have been calculated in QCD [18], and are of the same order as those of B_c [19]. As LHCb has a data sample larger than 3 fb^{-1} and is collecting even more data during Run 2, there is now a good opportunity to study Ξ_{cc}. One key issue left is to select the decay processes with the largest possibility of observing doubly charmed baryons.

In this work, we will systematically study the processes of Ξ_{cc}^{++} and Ξ_{cc}^{+} decays to find those with the

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largest branching fractions, which should be helpful for experimental searches for the doubly charmed baryons. The lowest-lying heavy particles can only decay weakly. We will analyze the color-allowed tree-operator dominated decay modes using the factorization ansatz. For other decay channels which are suppressed in the factorization scheme, the non-factorizable contributions might be significant and behave as long distance contributions. With a direct calculation in the rescattering mechanism, we will demonstrate that long-distance contributions are significantly enhanced for some decay modes with high experimental efficiencies. At the end, we will point out that instead of searching for the Ξ_{cc} using the SELEX discovery channels $\Xi_{cc}^+ \to \Lambda_c^+ K^- \pi^+$ and pD^+K^-, one should measure $\Xi_{cc}^{++} \to \Lambda_c^+ K^- \pi^+ \pi^+$ and $\Xi_c^+ \pi^+$ with the highest priority.

The branching fractions depend on the lifetimes of Ξ_{cc}^{++} and Ξ_{cc}^+ which, however, are predicted to be quite different in the literature. Predictions for the lifetime of Ξ_{cc}^+ vary from 53 fs to 250 fs, while those for Ξ_{cc}^{++} range from 185 fs to 670 fs [20–23], except for $\tau_{\Xi_{cc}^{++}}$=1550 fs in Ref. [24], which is too large compared to the lifetimes of singly charmed baryons. Despite the large ambiguity in the absolute lifetimes, it is expected that $\tau(\Xi_{cc}^{++}) \gg \tau(\Xi_{cc}^+)$ for their relative lifetimes, due to the effect of the destructive Pauli interference in the former. The ratio between their lifetimes is then

$$\mathcal{R}_\tau \equiv \frac{\tau_{\Xi_{cc}^+}}{\tau_{\Xi_{cc}^{++}}} = 0.25 \sim 0.37, \tag{1}$$

with small uncertainty in all these calculations [20–23]. The branching fractions of Ξ_{cc}^{++} decays should be relatively larger due to its longer lifetime, compared to those of Ξ_{cc}^+. Besides, particles with longer lifetimes can be better identified with high efficiency at the detectors. Thus, we recommend experimentalists to search for Ξ_{cc}^{++} before Ξ_{cc}^+.

2 Form factors

In the study of the exclusive modes of heavy hadron decays, the transition form factors are required in the calculations. The hadronic matrix elements of Ξ_{cc} decaying into the anti-triplet and sextet singly charmed baryons, i.e. $\mathcal{B}_c$=Ξ_c, Ξ_c', Λ_c and Σ_c, are expressed in terms of the form factors as

$$\begin{aligned}
&\langle \mathcal{B}_c(p_f)|J_\mu^w|\Xi_{cc}(p_i)\rangle \\
&= \bar{u}_f(p_f)\left[\gamma_\mu f_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{m_i} f_2(q^2) + \frac{q_\mu}{m_i} f_3(q^2)\right] u_i(p_i) \\
&- \bar{u}_f(p_f)\left[\gamma_\mu g_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{m_i} g_2(q^2) + \frac{q_\mu}{m_i} g_3(q^2)\right]\gamma_5 u_i(p_i),
\end{aligned} \tag{2}$$

where the initial and final baryons are all $\frac{1}{2}^+$ states, J_μ^w is the weak current in the relevant decays and q=p_i−p_f. In this work, the form factors are calculated in the light-front quark model (LFQM). The LFQM is a relativistic quark model under the light front approach, and has been successfully used to study the form factors of heavy meson and heavy baryon decays [25–27]. We adopt the diquark picture for the two spectator quarks [25, 26]. The diquark state with a charm quark and a light quark can be either a scalar (J^P=0^+), or an axial-vector state (1^+). Considering the wave functions of the relevant baryons, the hadronic matrix elements of Ξ_{cc} decaying into the anti-triplet and sextet singly charmed baryons are linear combinations of the transitions with the scalar and the axial-vector diquarks,

$$\begin{aligned}
\langle \mathcal{B}_c(\overline{\mathbf{3}})|J_\mu^w|\Xi_{cc}\rangle &= \frac{\sqrt{6}}{4}\langle J_\mu^w\rangle_{0^+} + \frac{\sqrt{6}}{4}\langle J_\mu^w\rangle_{1^+}, \\
\langle \mathcal{B}_c(\mathbf{6})|J_\mu^w|\Xi_{cc}\rangle &= -\frac{3\sqrt{2}}{4}\langle J_\mu^w\rangle_{0^+} + \frac{\sqrt{2}}{4}\langle J_\mu^w\rangle_{1^+}.
\end{aligned} \tag{3}$$

In the case with two identical quarks in the final state, for instance Σ_c^0, an overall factor of $\sqrt{2}$ has to be considered. The details of the calculations of the form factors in the LFQM can be seen in Ref. [29]. The results are given in Table 1, with the q^2 dependence of

$$F(q^2) = F(0)/\left(1 + \alpha\frac{q^2}{m_{\rm fit}^2} + \delta\frac{q^4}{m_{\rm fit}^4}\right), \tag{4}$$

where α=+1 for $g_2(q^2)$ with 1^+ diquarks as shown with stars in Table 1, and α=−1 for all the other form factors. Under the flavor $SU(3)$ symmetry, the form factors are related to each other between Ξ_{cc}^{++} and Ξ_{cc}^+ decays, and between $c \to s$ and $c \to d$ transitions as seen in Table 1. The uncertainties of the form factors can then be mostly cancelled in the relative branching fractions between decay channels.

Table 1. The transition form factors of Ξ_{cc} decaying into singly charmed baryons. The form factors with the scalar and the axial-vector diquarks are shown with 0^+ and 1^+ respectively.

	$\Xi_{cc}\to\Xi_c/\Xi_c'(0^+)$				$\Xi_{cc}\to\Xi_c/\Xi_c'(1^+)$			
	f_1	g_1	f_2	g_2	f_1	g_1	f_2	g_2^*
$F(0)$	0.75	0.62	−0.78	−0.08	0.74	−0.20	0.80	−0.02
$m_{\rm fit}$	1.84	2.16	1.67	1.29	1.58	2.10	1.62	1.62
δ	0.25	0.35	0.30	0.52	0.36	0.21	0.31	1.37
	$\Xi_{cc}\to\Lambda_c/\Sigma_c(0^+)$				$\Xi_{cc}\to\Lambda_c/\Sigma_c(1^+)$			
	f_1	g_1	f_2	g_2	f_1	g_1	f_2	g_2^*
$F(0)$	0.65	0.53	−0.74	−0.05	0.64	−0.17	0.73	−0.03
$m_{\rm fit}$	1.72	2.03	1.56	1.12	1.49	1.99	1.53	2.03
δ	0.27	0.38	0.32	1.10	0.37	0.23	0.32	2.62

3 Short-distance contribution dominated processes

With the form factors obtained above, we proceed to study the non-leptonic decays of Ξ_{cc}. The short-distance contributions in the external and internal W-emission amplitudes of two-body non-leptonic modes are calculated in the factorization approach manifested in the heavy quark limit. The amplitudes of Ξ_{cc} decaying into a singly charmed baryon and a light meson (M) can be expressed by the product of hadronic matrix elements

$$\mathcal{A}(\Xi_{cc}\to\mathcal{B}_c M)_{SD}=\lambda\langle M(q)|J^\mu|0\rangle\langle\mathcal{B}_c(p_f)|J^w_\mu|\Xi_{cc}(p_i)\rangle, \tag{5}$$

where $\lambda=\frac{G_F}{\sqrt{2}}V_{CKM}a_{1,2}(\mu)$, V_{CKM} denotes the product of the corresponding Cabibbo-Koboyashi-Maskawa matrix elements, and $a_1(\mu)=C_1(\mu)+C_2(\mu)/3$ for the external W-emission amplitudes and $a_2(\mu)=C_2(\mu)+C_1(\mu)/3$ for the internal W-emission ones, with $C_1(\mu)=1.21$ and $C_2(\mu)=-0.42$ at the scale of $\mu=m_c$ [32]. In this work M denotes a pseudoscalar meson (P) or a vector meson (V), with the hadronic matrix elements of decay constants as

$$\langle P(q)|J^\mu|0\rangle=\mathrm{i}f_P q^\mu, \langle V(q)|J^\mu|0\rangle=f_V m_V\epsilon^{\mu *} \tag{6}$$

In this work, we show the results of a few gold channels with the highest probability of being observed in experiments. More discussions on various processes can be found in Refs. [28–31]. According to Eq. (5), the relative branching fractions of the other processes compared to that of $\Xi_{cc}^{++}\to\Xi_c^+\pi^+$ are given as

$$\begin{aligned}&\mathcal{B}(\Xi_{cc}^+\to\Xi_c^0\pi^+)/\mathcal{B}(\Xi_{cc}^{++}\to\Xi_c^+\pi^+)=\mathcal{R}_\tau=0.25\sim0.37,\\&\mathcal{B}(\Xi_{cc}^{++}\to\Lambda_c^+\pi^+)/\mathcal{B}(\Xi_{cc}^{++}\to\Xi_c^+\pi^+)=0.056,\\&\mathcal{B}(\Xi_{cc}^{++}\to\Xi_c^+\ell^+\nu)/\mathcal{B}(\Xi_{cc}^{++}\to\Xi_c^+\pi^+)=0.71,\end{aligned} \tag{7}$$

The above relations are basically unambiguous, since the uncertainties from the transition form factors are mostly cancelled under the flavor $SU(3)$ symmetry. It is obvious that the branching fraction of $\Xi_{cc}^{++}\to\Xi_c^+\pi^+$ is the largest, compared to that of $\Xi_{cc}^{++}\to\Lambda_c^+\pi^+$ which is a Cabibbo-suppressed mode, and that of $\Xi_{cc}^+\to\Xi_c^0\pi^+$ due to the expected smaller lifetime of Ξ_{cc}^+. The semi-leptonic mode of $\Xi_{cc}^{++}\to\Xi_c^+ l^+\nu$ suffers a low efficiency of detection for the missing energies of neutrinos. Similarly, some other processes with possible larger branching fractions may lose lots of events at hadron colliders, due to neutral particles such as in $\Xi_c^+\rho^+(\to\pi^+\pi^0)$ and $\Xi_c'^+(\to\Xi_c^+\gamma)\pi^+$. The final state of $\Xi_c^+a_1^+(\to\pi^+\pi^+\pi^-)$ has two more tracks, reducing the efficiency of detection. Besides, the longer lifetimes of Ξ_{cc}^{++} and Ξ_c^+ compared to those of Ξ_{cc}^+ and Ξ_c^0, respectively, benefit higher efficiencies of the identification of the particles in experiments. Thus the $\Xi_{cc}^{++}\to\Xi_c^+\pi^+$ process is the best of the external W-emission processes to search for the doubly charmed baryons.

The absolute branching fraction of $\Xi_{cc}^{++}\to\Xi_c^+\pi^+$ is calculated to be

$$\mathcal{B}(\Xi_{cc}^{++}\to\Xi_c^+\pi^+)=\left(\frac{\tau_{\Xi_{cc}^{++}}}{300\,\mathrm{fs}}\right)\times7.2\%. \tag{8}$$

This result is given by comparing the lifetime of Ξ_{cc}^{++} with 300 fs, which is in the range of the predictions. Even if considering the uncertainties of the transition form factors and the lifetime, the branching fraction of this process is of the order of percent, which is large enough for measurements.

To measure $\Xi_{cc}^{++}\to\Xi_c^+\pi^+$, Ξ_c^+ can be reconstructed using the mode $\Xi_c^+\to\mathrm{pK}^-\pi^+$ at hadron colliders with all the charged particles in the final state. The absolute branching fraction of this process has never been directly measured, but the relative branching ratio was measured as $\mathcal{B}(\Xi_c^+\to\mathrm{p}\overline{\mathrm{K}}^{*0})/\mathcal{B}(\Xi_c^+\to\mathrm{pK}^-\pi^+)=0.54\pm0.10$ [33]. Besides, the relation $\mathcal{A}(\Xi_c^+\to\mathrm{p}\overline{\mathrm{K}}^{*0})=\mathcal{A}(\Lambda_c^+\to\Sigma^+\mathrm{K}^{*0})$ holds under U-spin symmetry. With the measurement of $\mathcal{B}(\Lambda_c^+\to\Sigma^+\mathrm{K}^{*0})=(0.36\pm0.10)\%$ [34], the branching fraction is

$$\mathcal{B}(\Xi_c^+\to\mathrm{pK}^-\pi^+)=(2.2\pm0.8)\%. \tag{9}$$

The relatively larger branching fraction of this Cabibbo-suppressed mode is induced by the larger phase space of $\Xi_c^+\to\mathrm{p}\overline{\mathrm{K}}^{*0}$ and the longer lifetime of Ξ_c^+. The main uncertainty in Eq. (9) arises from the branching fraction of $\Lambda_c^+\to\Sigma^+\mathrm{K}^{*0}$ and the ratio between $\Xi_c^+\to\mathrm{p}\overline{\mathrm{K}}^{*0}$ and $\Xi_c^+\to\mathrm{pK}^-\pi^+$, which may be measured by BESIII, Belle II and LHCb with higher precision. Considering the relatively large value of $\mathcal{B}(\Xi_c^+\to\mathrm{pK}^-\pi^+)$ within the 1σ range, we suggest to measure the process of $\Xi_{cc}^{++}\to\Xi_c^+\pi^+$ with Ξ_c^+ reconstructed by the final state $\mathrm{pK}^-\pi^+$.

4 Long-distance contribution dominated processes

In the factorization approach, only the factorizable contributions are taken into account. For the color-allowed tree-operator dominated channels, the non-factorizable contributions are expected to be small. For the color-suppressed processes with a tiny Wilson coefficient a_2, the decay widths are likely to be underestimated in the factorization framework. For instance, the branching fractions of the internal W-emission decays of $\Xi_{cc}^{++}\to\Sigma_c^{++}(2455)\overline{\mathrm{K}}^{*0}$ and $\Xi_{cc}^+\to\Lambda_c^+\overline{\mathrm{K}}^{*0}$ are predicted to be of the order of 10^{-5}, due to $a_2(\mu)\approx-0.02$.

However, the long-distance contributions are usually significantly enhanced in charmed meson decays, which can be described well by the rescattering mechanism of the final-state-interaction effects [35–38]. The rescattering mechanism in the heavy-flavor-baryon decays was only considered in Ref. [39] to study the Cabibbo-suppressed decays of $\Lambda_c^+\to\mathrm{p}\pi^0$ and $n\pi^+$, whose results

have not been directly manifested so far, but are consistent with the upper limit recently measured by BESIII [40]. In doubly heavy flavor baryon decays, the long-distance contributions have never been considered. In this work we first calculate the rescattering effects in two-body non-leptonic Ξ_{cc} decays for the internal W-emission and W-exchange amplitudes, and then find some other processes with large branching fractions.

The absorptive part of the amplitudes is obtained by the optical theorem [41], summing over all possible amplitudes of $\Xi_{cc}(p_i)$ decaying into the states $\{p_k\}$, followed by the rescattering of $\{p_k\}$ into the final state $\mathcal{B}_c(p_f)M(q)$,

$$\mathrm{Abs}\mathcal{M}(p_i\to \mathrm{p_f q})=\frac{1}{2}\sum_{\mathrm{j}}\left(\prod_{\mathrm{k}=1}^{\mathrm{j}}\int\frac{\mathrm{d}^3\mathrm{p_k}}{(2\pi)^3 2\mathrm{E_k}}\right)(2\pi)^4$$

$$\times\delta^4(p_f+q-\sum_{k=1}^{j}p_k)\mathcal{M}(p\to\{p_k\})T^*(p_f q\to\{p_k\}). \quad (10)$$

One typical rescattering diagram is given in Fig. 1, taking as example the t-channel triangle diagram of $\Xi_{cc}^{++}\to\Xi_c^{(\prime)+}\rho^+\to\Sigma_c^{++}\overline{\mathrm{K}}^{*0}$ via quark exchange.

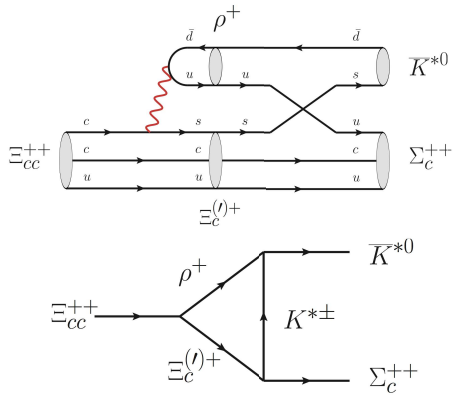

Fig. 1. (color online) The t-channel rescattering diagram of $\Xi_{cc}^{++}\to\Xi_c^{(\prime)+}\rho^+\to\Sigma_c^{++}\overline{\mathrm{K}}^{*0}$. The rescattering is induced by the quark exchanges, shown in the top figure. The bottom figure is the diagram at the hadron level.

The rescattering amplitudes are calculated using the effective Lagrangian [42–45]. The hadronic strong coupling constants are related to each other under the flavor $SU(3)$ symmetry, and the chiral and heavy quark symmetries [42–44], with the values taken from Refs. [41, 46–52]. The effective Lagrangian and the strong coupling constants are given in the Appendix. Most of the uncertainties will then be cancelled in the relative branching ratios. The results of the rescattering amplitudes depend on the form factor $F(t,m)$ which describes the off-shell effect of the exchanged particle. It is parametrized as $F(t,m)=(\Lambda^2-m^2)/(\Lambda^2-t)$ [41], with the cutoff $\Lambda=m+\eta\Lambda_{\mathrm{QCD}}$, m and t being the mass and the momentum squared of the exchanged particle, respectively, and Λ_{QCD} taken as 330 MeV. The free parameter η cannot be calculated from first principles. In this work, we take η varying in the range from 1.0 to 2.0, as found in Ref. [41]. The dependence of $F(t,m)$ on η is plotted in Fig. 2.

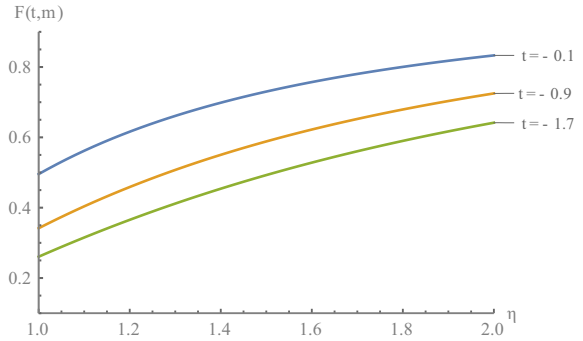

Fig. 2. (color online) The form factor $F(t,m)$ as a function of η.

The relative branching fractions of some processes dominated by the long-distance contributions compared to that of $\Xi_{cc}^{++}\to\Sigma_c^{++}(2455)\overline{\mathrm{K}}^{*0}$ are shown in Table 2. The results of the relative branching fractions are less ambiguous in theory, since the uncertainties from the effective hadronic strong coupling constants and from the transition form factors are mainly cancelled due to the flavor $SU(3)$ symmetry and the chiral and heavy quark symmetries as discussed before. The absolute branching fractions depend heavily on the values of the parameter η, as seen in the top plot of Fig. 3, taking as examples $\mathcal{B}(\Xi_{cc}^{++}\to\Sigma_c^{++}(2455)\overline{\mathrm{K}}^{*0})$ and $\mathcal{B}(\Xi_{cc}^{++}\to\mathrm{pD}^{*+})$ as functions of η. In the bottom of Fig. 3, we plot the ratio $\mathcal{B}(\Xi_{cc}^{++}\to\mathrm{pD}^{*+})/\mathcal{B}(\Xi_{cc}^{++}\to\Sigma_c^{++}\overline{\mathrm{K}}^{*0})$ as a function of η. The ratio of branching fractions is insensitive to η. Therefore, the theoretical uncertainties are under control for the relative branching fractions. From Table 2, it is obvious that $\Xi_{cc}^{++}\to\Sigma_c^{++}(2455)\overline{\mathrm{K}}^{*0}$ has the largest branching fraction, which is useful for experimental measurements.

Table 2. Branching fractions of Ξ_{cc}^{++} and Ξ_{cc}^{+} decays with the long-distance contributions, relative to that of $\Xi_{cc}^{++}\to\Sigma_c^{++}(2455)\overline{\mathrm{K}}^{*0}$.

baryons	modes	$\mathcal{B}_{\mathrm{LD}}$
$\Xi_{cc}^{++}(ccu)$	$\Sigma_c^{++}(2455)\overline{\mathrm{K}}^{*0}$	defined as 1
	pD^{*+}	0.04
	pD^{+}	0.0008
$\Xi_{cc}^{+}(ccd)$	$\Lambda_c^+\overline{\mathrm{K}}^{*0}$	$(\mathcal{R}_\tau/0.3)\times0.22$
	$\Sigma_c^{++}(2455)\mathrm{K}^-$	$(\mathcal{R}_\tau/0.3)\times0.01$
	$\Xi_c^+\rho^0$	$(\mathcal{R}_\tau/0.3)\times0.04$
	$\Lambda\mathrm{D}^+$	$(\mathcal{R}_\tau/0.3)\times0.004$
	pD^0	$(\mathcal{R}_\tau/0.3)\times0.001$

In the process $\Xi_{cc}^{++}\to\Sigma_c^{++}(2455)\overline{K}^{*0}$, the dominant rescattering amplitude is $\Xi_{cc}^{++}\to\Xi_c^{(\prime)+}\rho^+\to\Sigma_c^{++}\overline{K}^{*0}$ with exchange of $K^{*\pm}$, depicted in Fig. 1. Considering some other triangle diagrams, including the t-channel rescattering by $\Xi_c^{(\prime)+}\pi^+$ with exchange of $K^{\pm}$, and u-channels rescattering by $\Xi_c^{(\prime)+}\pi^+$ and $\Xi_c^{(\prime)+}\rho^+$ with exchange of Λ_c^+ or Σ_c^+, the absolute branching fraction is

$$\mathcal{B}(\Xi_{cc}^{++}\to\Sigma_c^{++}(2455)\overline{K}^{*0})=\left(\frac{\tau_{\Xi_{cc}^{++}}}{300\,\text{fs}}\right)\times(3.8\sim24.6)\%, \tag{11}$$

where the range corresponds to the value of η varying between 1.0 and 2.0. Compared to the short-distance results of $\Xi_{cc}^{++}\to\Sigma_c^{++}(2455)\overline{K}^{*0}$ as $\mathcal{O}(10^{-5})$, the long-distance contributions in the doubly charmed baryon decays are significantly enhanced. The branching fractions of $\Xi_{cc}^{++}\to\Xi_c^+\rho^+$ and $\Xi_c^{\prime+}\rho^+$ are 12.6% and 17.4% respectively, which are large enough to lead to a result for $\Xi_{cc}^{++}\to\Sigma_c^{++}(2455)\overline{K}^{*0}$ of the order of percent. In the charmed meson decays, the large-N_c approach gives a good description of the internal W-emission contributions, which amounts to $|a_2^{\text{eff}}(\mu_c)|\approx C_2(\mu_c)\sim-0.5$ [53]. With this value, the branching fraction of this process is then 4.6%, which is in the range of Eq. (11). So the results considering the rescattering mechanism for long-distance contributions are trustworthy.

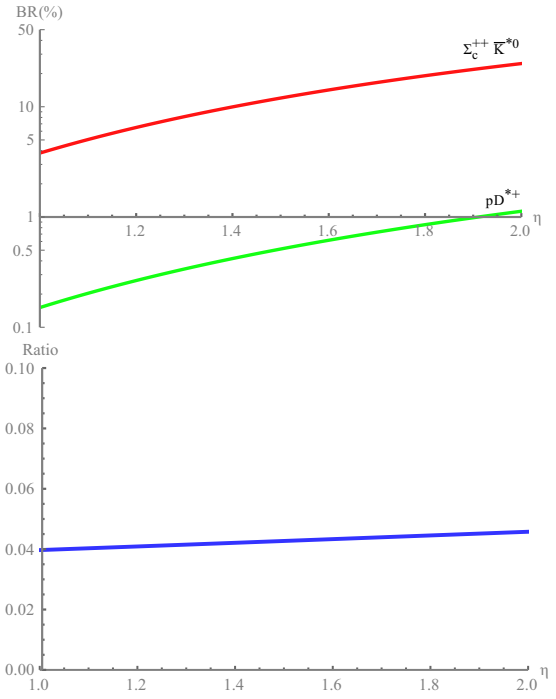

Fig. 3. (color online) The absolute branching fractions of $\Xi_{cc}^{++}\to\Sigma_c^{++}\overline{K}^{*0}$ and pD^{*+} (top) and the ratio of $\mathcal{B}(\Xi_{cc}^{++}\to pD^{*+})/\mathcal{B}(\Xi_{cc}^{++}\to\Sigma_c^{++}\overline{K}^{*0})$ (bottom) as functions of η.

$\Xi_{cc}^{++}\to\Sigma_c^{++}(2455)\overline{K}^{*0}$ is actually a four-body process with the strong decays of $\Sigma_c^{++}\to\Lambda_c^+\pi^+$ and $\overline{K}^{*0}\to K^-\pi^+$. In the charmed meson decays, the resonant contributions almost saturate the decay widths [54]. The final-state particles are not very energetic in charm decays, and hence easily located in the momentum range of resonances. Thus the resonant contributions can indicate the key physics. For the four-body process $\Xi_{cc}^{++}\to\Lambda_c^+K^-\pi^+\pi^+$, there are many low-lying-resonance contributions, such as $\Sigma_c^{++}(2455)$ and $\Sigma_c^{++}(2520)$ for $\Lambda_c^+\pi^+$, and $\overline{K}^{*0}$ and $(K\pi)_{\text{S-wave}}$ for $K^-\pi^+$. Recalling that $\mathcal{B}(\Xi_{cc}^{++}\to\Sigma_c^{++}(2455)\overline{K}^{*0})$ is as large as shown in Eq. (11), the branching fraction of $\Xi_{cc}^{++}\to\Lambda_c^+K^-\pi^+\pi^+$ will be expected to be a few percent, or even reach $\mathcal{O}(10\%)$. With Λ_c^+ reconstructed by $pK^-\pi^+$, the process $\Xi_{cc}^{++}\to\Lambda_c^+K^-\pi^+\pi^+$ can be used to search for the doubly charmed baryon.

Apart from the above four-body decay mode, the branching fraction of $\Xi_{cc}^+\to\Lambda_c^+K^-\pi^+$ should be considerable, contributed by $\Lambda_c^+\overline{K}^{*0}$ and $\Sigma_c^{++}(2455)K^-$. This process is just the one used by the SELEX experiment to report the first would-be evidence of Ξ_{cc}^+ [5]. However, no significant signal was observed by the LHCb experiment using this channel [11]. We find the process $\Xi_{cc}^{++}\to\Lambda_c^+K^-\pi^+\pi^+$ is better than $\Xi_{cc}^+\to\Lambda_c^+K^-\pi^+$ for the searches for doubly charmed baryons, for the following reasons. For the dominant resonant contributions in these two processes, the branching fraction of $\Xi_{cc}^{++}\to\Sigma_c^{++}(2455)\overline{K}^{*0}$ is larger than that of $\Xi_{cc}^+\to\Lambda_c^+\overline{K}^{*0}$ by a factor of about five, due to the predicted value of $\mathcal{R}_\tau\sim0.3$ with a small uncertainty, seen in Eq. (1). As explained before, the efficiency of identifying Ξ_{cc}^{++} is larger than that of Ξ_{cc}^+ at LHCb by a factor around the lifetime ratio, in the range of their predicted lifetimes. Even though $\Xi_{cc}^{++}\to\Lambda_c^+K^-\pi^+\pi^+$ suffers a lower efficiency in detection by a few factors, due to one more track than $\Xi_{cc}^+\to\Lambda_c^+K^-\pi^+$, it can still be expected that there would be more signal yields in the former process. For the other discovery channel by the SELEX experiment, $\Xi_{cc}^+\to pD^+K^-$, there are no low-lying-resonance contributions. In the mode $\Xi_{cc}^+\to\Lambda D^+$, whose branching ratio is small, the Λ state is below the pK^- threshold, while the higher excited resonances would be more difficult to produce. Therefore, the process $\Xi_{cc}^{++}\to\Lambda_c^+K^-\pi^+\pi^+$ is the best of the long-distance contribution dominated processes for the searches for doubly charmed baryons.

In addition to the study of the ground states of doubly charmed baryons, the suggested processes should be useful to search for excited states below the charm-meson-charm-baryon thresholds. Such particles strongly or radiatively decay into the ground states, which would be reconstructed in experiments by the most favorable modes found in this work. Besides, the long-distance contributions, for which we found large and important

Ξ_{cc} decays, should also be considered in the studies on the search for the discovery channels of other heavy particles, such as bottom-charm baryons and stable open flavor tetraquarks and pentaquarks.

5 Summary

We have systematically studied the weak decays of Ξ_{cc}^{++} and Ξ_{cc}^{+} and recommend the processes $\Xi_{cc}^{++}\to\Lambda_c^+K^-\pi^+\pi^+$ and $\Xi_{cc}^{+}\pi^+$ as the most favorable decay modes for searches for doubly charmed baryons in experiments. The channels $\Xi_{cc}^{+}\to\Lambda_c^+K^-\pi^+$ and pD^+K^- used by the SELEX and LHCb experiments are not as good as the above two Ξ_{cc}^{++} decay processes. The short-distance contributions of the decay amplitudes are calculated under the factorization approach. The long-distance contributions are first studied in the double-charm-baryon decays, considering the rescattering mechanism. It is found that the long-distance contributions are significantly enhanced and are essential for the favorable mode $\Xi_{cc}^{++}\to\Lambda_c^+K^-\pi^+\pi^+$. Our suggestions are based on the analysis of the relative branching fractions between decay modes, which is less ambiguous since the theoretical uncertainties are mainly cancelled by the flavor symmetries. The absolute branching fractions of $\Xi_{cc}^{++}\to\Lambda_c^+K^-\pi^+\pi^+$ and $\Xi_c^+\pi^+$ are estimated to be a few percent, or even reach the order of 10%, which are large enough for experimental measurements[1)].

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Appendix A

The effective Lagrangians used in the rescattering mechanism are [42–45]

$$
\begin{aligned}
&\mathcal{L}_{eff}=\mathcal{L}_{\pi hh}+\mathcal{L}_{\rho hh}+\mathcal{L}_{\pi\mathcal{B}\mathcal{B}}+\mathcal{L}_{\rho\mathcal{B}\mathcal{B}}+\mathcal{L}_{\rho\pi\pi}+\mathcal{L}_{\rho\rho\rho}+\mathcal{L}_{\rho DD}+\mathcal{L}_{\pi D^*D}+\mathcal{L}_{\rho D^*D^*},\\
&\mathcal{L}_{\pi hh}=g_{\pi\mathcal{B}_6\mathcal{B}_6}Tr[\bar{\mathcal{B}}_6 i\gamma_5\Pi\mathcal{B}_6]+g_{\pi\mathcal{B}_{\bar3}\mathcal{B}_{\bar3}}Tr[\bar{\mathcal{B}}_{\bar3} i\gamma_5\Pi\mathcal{B}_{\bar3}]+\{g_{\pi\mathcal{B}_6\mathcal{B}_{\bar3}}Tr[\bar{\mathcal{B}}_6 i\gamma_5\Pi\mathcal{B}_{\bar3}]+h.c.\},\\
&\mathcal{L}_{\rho hh}=f_{1\rho\mathcal{B}_6\mathcal{B}_6}Tr[\bar{\mathcal{B}}_6\gamma_\mu V^\mu\mathcal{B}_6]+\frac{f_{2\rho\mathcal{B}_6\mathcal{B}_6}}{2m_6}Tr[\bar{\mathcal{B}}_6\sigma_{\mu\nu}\partial^\mu V^\nu\mathcal{B}_6]\\
&\qquad+f_{1\rho\mathcal{B}_{\bar3}\mathcal{B}_{\bar3}}Tr[\bar{\mathcal{B}}_{\bar3}\gamma_\mu V^\mu\mathcal{B}_{\bar3}]+\frac{f_{2\rho\mathcal{B}_{\bar3}\mathcal{B}_{\bar3}}}{2m_{\bar3}}Tr[\bar{\mathcal{B}}_{\bar3}\sigma_{\mu\nu}\partial^\mu V^\nu\mathcal{B}_{\bar3}]\\
&\qquad+\{f_{1\rho\mathcal{B}_6\mathcal{B}_{\bar3}}Tr[\bar{\mathcal{B}}_6\gamma_\mu V^\mu\mathcal{B}_{\bar3}]+\frac{f_{2\rho\mathcal{B}_6\mathcal{B}_{\bar3}}}{m_6+m_{\bar3}}Tr[\bar{\mathcal{B}}_6\sigma_{\mu\nu}\partial^\mu V^\nu\mathcal{B}_{\bar3}]+h.c.\},\\
&\mathcal{L}_{\pi\mathcal{B}\mathcal{B}}=g_{\pi\mathcal{B}\mathcal{B}}Tr[\bar{\mathcal{B}}i\gamma_5\Pi\mathcal{B}],\\
&\mathcal{L}_{\rho\mathcal{B}\mathcal{B}}=f_{1\rho\mathcal{B}\mathcal{B}}Tr[\bar{\mathcal{B}}\gamma_\mu V^\mu\mathcal{B}]+\frac{f_{2\rho\mathcal{B}\mathcal{B}}}{2m_{\mathcal{B}}}Tr[\bar{\mathcal{B}}\sigma_{\mu\nu}\partial^\mu V^\nu\mathcal{B}],\\
&\mathcal{L}_{\rho\pi\pi}=\frac{ig_{\rho\pi\pi}}{\sqrt2}Tr[V^\mu[\Pi,\partial_\mu\Pi]],\\
&\mathcal{L}_{\rho\rho\rho}=\frac{ig_{\rho\rho\rho}}{\sqrt2}Tr[(\partial_\nu V_\mu-\partial_\mu V_\nu)V^\mu V^\nu]=\frac{ig_{\rho\rho\rho}}{\sqrt2}Tr[(\partial_\nu V_\mu V^\mu-V^\mu\partial_\nu V_\mu)V^\nu],\\
&\mathcal{L}_{\rho DD}=-ig_{\rho DD}(D_i\partial_\mu D^{j\dagger}-\partial_\mu D_i D^{j\dagger})(V^\mu)^i_j,\\
&\mathcal{L}_{\pi D^*D}=-g_{\pi D^*D}(D^i\partial^\mu\Pi_{ij}D^{*j\dagger}_\mu+D^{*i}_\mu\partial^\mu\Pi_{ij}D^{j\dagger}),\\
&\mathcal{L}_{\rho D^*D^*}=ig_{\rho D^*D^*}(D^{*\nu}_i\partial_\mu D^{*j\dagger}_\nu-\partial_\mu D^{*\nu}_i D^{*j\dagger}_\nu)(V^\mu)^i_j+4if_{\rho D^*D^*}D^{*\dagger}_{i\mu}(\partial^\mu V^\nu-\partial^\nu V^\mu)^i_j D^{*j}_\nu.
\end{aligned}
\tag{A1}
$$

where the corresponding Π, V, $\mathcal{B}$ and $\mathcal{B}_6$, $\mathcal{B}_{\bar3}$ respectively represent the matrices

$$
\Pi=\begin{pmatrix}\frac{\pi^0}{\sqrt2}+\frac{\eta}{\sqrt6} & \pi^+ & K^+\\ \pi^- & -\frac{\pi^0}{\sqrt2}+\frac{\eta}{\sqrt6} & K^0\\ K^- & \bar K^0 & -\sqrt{\frac23}\eta\end{pmatrix},\quad
\mathcal{B}_6=\begin{pmatrix}\Sigma_c^{++} & \frac{1}{\sqrt2}\Sigma_c^+ & \frac{1}{\sqrt2}\Xi_c^{\prime+}\\ \frac{1}{\sqrt2}\Sigma_c^+ & \Sigma_c^0 & \frac{1}{\sqrt2}\Xi_c^{\prime0}\\ \frac{1}{\sqrt2}\Xi_c^{\prime+} & \frac{1}{\sqrt2}\Xi_c^{\prime0} & \Omega_c\end{pmatrix},
$$

$$
V=\begin{pmatrix}\frac{\rho^0}{\sqrt2}+\frac{\omega}{\sqrt2} & \rho^+ & K^{*+}\\ \rho^- & -\frac{\rho^0}{\sqrt2}+\frac{\omega}{\sqrt2} & K^{*0}\\ K^{*-} & \bar K^{*0} & \phi\end{pmatrix},\quad
\mathcal{B}_{\bar3}=\begin{pmatrix}0 & \Lambda_c^+ & \Xi_c^+\\ -\Lambda_c^+ & 0 & \Xi_c^0\\ -\Xi_c^+ & -\Xi_c^0 & 0\end{pmatrix},\quad
\mathcal{B}=\begin{pmatrix}\frac{\Sigma^0}{\sqrt2}+\frac{\Lambda}{\sqrt6} & \Sigma^+ & p\\ \Sigma^- & -\frac{\Sigma^0}{\sqrt2}+\frac{\Lambda}{\sqrt6} & n\\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt6}\Lambda\end{pmatrix}.
\tag{A2}
$$

1) Note added: very recently, the LHCb collaboration reported the discovery of Ξ_{cc}^{++} with the final state $\Lambda_c^+K^-\pi^+\pi^+$ [55].

According to the generalized form of baryons coupled with mesons in Eq. (A1), we extend to the vertex $\mathcal{B}_c\mathcal{B}D$ and $\mathcal{B}_c\mathcal{B}D^*$, and write the Lagrangian as

$$\begin{aligned}
\mathcal{L}_{\Lambda_c N D_q} &= g_{\Lambda_c N D_q}(\bar{\Lambda}_c i\gamma_5 D_q N + h.c.),\\
\mathcal{L}_{\Lambda_c N D_q^*} &= f_{1\Lambda_c N D_q^*}(\bar{\Lambda}_c \gamma_\mu D_q^{*\mu} N + h.c.)\\
&\quad + \frac{f_{2\Lambda_c N D_q^*}}{m_{\Lambda_c}+m_N}(\bar{\Lambda}_c \sigma_{\mu\nu}\partial^\mu D_q^{*\nu} N + h.c.)\\
\mathcal{L}_{\Sigma_c N D_q} &= g_{\Sigma_c N D_q}(\bar{\Sigma}_c i\gamma_5 D_q N + h.c.),\\
\mathcal{L}_{\Sigma_c N D_q^*} &= f_{1\Sigma_c N D_q^*}(\bar{\Sigma}_c \gamma_\mu D_q^{*\mu} N + h.c.)\\
&\quad + \frac{f_{2\Sigma_c N D_q^*}}{m_{\Sigma_c}+m_N}(\bar{\Sigma}_c \sigma_{\mu\nu}\partial^\mu D_q^{*\nu} N + h.c.).
\end{aligned} \tag{A3}$$

where N denotes baryons belong to the octet baryon matrix $\mathcal{B}$.

The strong coupling constants are taken from the literature [41, 46–52], and listed in Tables A1, A2 and A3.

Table A1. The strong coupling constants for VPP, VVV, $\mathcal{B}_c\mathcal{B}_cP$, $\mathcal{B}\mathcal{B}P$ and $\mathcal{B}_c\mathcal{B}D$.

vertex	g	vertex	g	vertex	g
$\rho\to\pi\pi$	6.05	$K^*\to K\pi$	4.60	$K^*\to K^*\rho$	5.22
$\Xi_c^0\to\Xi_c^+\pi^-$	0.70	$\Xi_c^0\to\Xi_c'^+\pi^-$	3.10	$\Xi_c^+\to\Xi_c^0\pi^+$	0.99
$\Xi_c^+\to\Xi_c'^0\pi$	4.38	$\Xi_c'^+\to\Xi_c'^0\pi$	5.66	$\Sigma_c^{++}\to\Xi_c^+K^+$	−6.5
$\Xi_c'^+\to\Sigma_c^{++}K^-$	9.0	$\Sigma_c^{++}\to\Sigma_c^+\pi^+$	8.0	$\Sigma_c^{++}\to\Lambda_c^+\pi^+$	−6.36
$p\to n\pi^+$	12.0	$\Lambda_c^+\to pD^0$	4.82	$\Sigma_c^+\to pD^0$	3.78
$\Lambda_c^+\to nD^+$	4.82	$\Sigma_c^+\to nD^+$	3.78	$\Lambda_c^+\to\Xi_c^0K^+$	−0.9
$\Lambda_c^+\to\Xi_c'^0K^+$	4.38	$\Lambda_c^+\to\Sigma_c^0\pi^+$	6.5	$\Sigma_c^+\to\Xi_c^0K^+$	−6.5
$\Sigma_c^+\to\Xi_c'^0K^+$	5.66	$\Lambda^0\to\Sigma^-\pi^+$	10.0	$\Xi_c^0\to\Lambda^0D^0$	4.82
$\Xi_c'^0\to\Lambda^0D^0$	3.78	$\Xi_c^0\to\Sigma^-D^+$	4.82	$\Xi_c'^0\to\Sigma^-D^+$	3.78

Table A2. The strong coupling constants for $\mathcal{B}_c\mathcal{B}_cV$, $\mathcal{B}\mathcal{B}V$ and $\mathcal{B}_c\mathcal{B}D^*$.

vertex	f_1	f_2	vertex	f_1	f_2
$\Sigma_c^{++}\to\Xi_c^+K^{*+}$	−3.11	−18.4	$\Sigma_c^{++}\to\Xi_c'^+K^{*+}$	5.0	30
$\Xi_c^+\to\Lambda_c^+\overline{K}^{*0}$	4.6	6.0	$\Xi_c'^+\to\Lambda_c^+\overline{K}^{*0}$	−2.2	−13.0
$\Sigma_c^{++}\to\Lambda_c^+\rho^+$	−2.6	−16.0	$\Sigma_c^+\to\Xi_c^+\overline{K}^{*0}$	−2.2	−13.0
$\Sigma_c^+\to\Xi_c'^+\overline{K}^{*0}$	2.82	19.1	$\Sigma_c^{++}\to\Sigma_c^+\rho^+$	4.0	27.0
$\Lambda_c^+\to pD^{*0}$	2.05	7.78	$\Sigma_c^+\to pD^{*0}$	11.21	4.64
$p\to n\rho^+$	−2.3	36.1	$\Lambda_c^+\to nD^{*+}$	2.05	7.78
$\Sigma_c^+\to nD^{*+}$	11.21	4.64	$\Xi_c^0\to\Lambda_c^+K^{*-}$	−4.6	−6.0
$\Xi_c'^0\to\Lambda_c^+K^{*-}$	2.12	15.6	$\Sigma_c^0\to\Xi_c^0K^{*0}$	−3.11	−18.4
$\Sigma_c^0\to\Xi_c'^0K^{*0}$	5.0	30.0	$\Sigma_c^+\to\Lambda_c^+\rho^-$	2.6	16.0
$\Xi_c^0\to\Xi_c^0\rho^0$	−6.0	−7.5	$\Xi_c'^0\to\Xi_c^0\rho^0$	2.12	15.6
$\Xi_c'^0\to\Xi_c'^0\rho^0$	−2.5	−16.0	$\Xi_c^+\to\Xi_c^0\rho^+$	8.5	10.6
$\Xi_c'^0\to\Xi_c'^+\rho^-$	2.12	15.6	$\Lambda^0\to\Sigma^-\rho^+$	2.0	12.3
$\Xi_c^0\to\Lambda^0D^{*0}$	2.05	7.78	$\Xi_c'^0\to\Lambda^0D^{*0}$	11.21	4.64

Table A3. The strong coupling constants for $D^*D\Pi$, DDV and D^*D^*V.

vertex	g	vertex	g	vertex	g	f
$D^*\to D\pi$	17.9	$D\to D\rho$	3.69	$D^*\to D^*\rho$	3.69	4.61

References

1 E. Klempt and J. M. Richard, Rev. Mod. Phys., **82**: 1095 (2010) doi: 10.1103/RevModPhys.82.1095 [arXiv: 0901.2055 [hep-ph]]

2 V. Crede and W. Roberts, Rept. Prog. Phys., **76**: 076301 (2013) doi: 10.1088/0034-4885/76/7/076301 [arXiv: 1302.7299 [nucl-ex]]

3 H. Y. Cheng, Front. Phys. (Beijing), **10**(6): 101406 (2015) doi: 10.1007/s11467-015-0483-z

4 H. X. Chen, W. Chen, X. Liu, Y. R. Liu, and S. L. Zhu, Rept. Prog. Phys., **80**(7): 076201 (2017) doi: 10.1088/1361-6633/aa6420 [arXiv: 1609.08928 [hep-ph]]

5 M. Mattson et al (SELEX Collaboration), Phys. Rev. Lett., **89**: 112001 (2002) doi: 10.1103/PhysRevLett.89.112001 [hep-ex/0208014]

6 A. Ocherashvili et al (SELEX Collaboration), Phys. Lett. B, **628**: 18 (2005) doi: 10.1016/j.physletb.2005.09.043 [hep-ex/0406033]

7 S. P. Ratti, Nucl. Phys. Proc. Suppl., **115**: 33 (2003). doi: 10.1016/S0920-5632(02)01948-5

8 B. Aubert et al (BaBar Collaboration), Phys. Rev. D, **74**: 011103 (2006) doi: 10.1103/PhysRevD.74.011103 [hep-ex/0605075]

9 R. Chistov et al (Belle Collaboration), Phys. Rev. Lett., **97**: 162001 (2006) doi: 10.1103/PhysRevLett.97.162001 [hep-

ex/0606051]
10 Y. Kato et al (Belle Collaboration), Phys. Rev. D, **89**(5): 052003 (2014) doi: 10.1103/PhysRevD.89.052003 [arXiv: 1312.1026 [hep-ex]]
11 R. Aaij et al (LHCb Collaboration), JHEP, **1312**: 090 (2013) doi: 10.1007/JHEP12(2013)090 [arXiv: 1310.2538 [hep-ex]]
12 R. Lewis, N. Mathur, and R. M. Woloshyn, Phys. Rev. D, **64**: 094509 (2001) doi: 10.1103/PhysRevD.64.094509 [hep-ph/0107037]
13 J. M. Flynn et al (UKQCD Collaboration), JHEP, **0307**: 066 (2003) doi: 10.1088/1126-6708/2003/07/066 [hep-lat/0307025]
14 L. Liu, H. W. Lin, K. Orginos, and A. Walker-Loud, Phys. Rev. D, **81**: 094505 (2010) doi: 10.1103/PhysRevD.81.094505 [arXiv: 0909.3294 [hep-lat]]
15 C. Alexandrou, J. Carbonell, D. Christaras, V. Drach, M. Gravina, and M. Papinutto, Phys. Rev. D, **86**: 114501 (2012) doi: 10.1103/PhysRevD.86.114501 [arXiv: 1205.6856 [hep-lat]]
16 R. A. Briceno, H. W. Lin, and D. R. Bolton, Phys. Rev. D, **86**: 094504 (2012) doi: 10.1103/PhysRevD.86.094504 [arXiv: 1207.3536 [hep-lat]]
17 C. Alexandrou, V. Drach, K. Jansen, C. Kallidonis, and G. Koutsou, Phys. Rev. D, **90**(7): 074501 (2014) doi: 10.1103/PhysRevD.90.074501 [arXiv: 1406.4310 [hep-lat]]
18 J. W. Zhang, X. G. Wu, T. Zhong, Y. Yu, and Z. Y. Fang, Phys. Rev. D, **83**: 034026 (2011) doi: 10.1103/PhysRevD.83.034026 [arXiv: 1101.1130 [hep-ph]]
19 C. H. Chang, C. F. Qiao, J. X. Wang, and X. G. Wu, Phys. Rev. D, **71**: 074012 (2005) doi: 10.1103/PhysRevD.71.074012 [hep-ph/0502155]
20 M. Karliner and J. L. Rosner, Phys. Rev. D, **90**(9): 094007 (2014) doi: 10.1103/PhysRevD.90.094007 [arXiv: 1408.5877 [hep-ph]]
21 V. V. Kiselev and A. K. Likhoded, Phys. Usp., **45**: 455 (2002) [Usp. Fiz. Nauk **172**: 497 (2002)] doi: 10.1070/PU2002v045n05ABEH000958 [hep-ph/0103169]
22 C. H. Chang, T. Li, X. Q. Li, and Y. M. Wang, Commun. Theor. Phys., **49**: 993 (2008) doi: 10.1088/0253-6102/49/4/38 [arXiv: 0704.0016 [hep-ph]]
23 A. I. Onishchenko, hep-ph/0006295
24 B. Guberina, B. Melic, and H. Stefancic, Eur. Phys. J. C, **9**: 213 (1999) [Eur. Phys. J. C, **13**: 551 (2000)] doi: 10.1007/s100529900039, 10.1007/s100520050525 [hep-ph/9901323]
25 H. W. Ke, X. Q. Li, and Z. T. Wei, Phys. Rev. D, **77**: 014020 (2008) doi: 10.1103/PhysRevD.77.014020 [arXiv: 0710.1927 [hep-ph]]
26 H. W. Ke, X. H. Yuan, X. Q. Li, Z. T. Wei, and Y. X. Zhang, Phys. Rev. D, **86**: 114005 (2012) doi: 10.1103/PhysRevD.86.114005 [arXiv: 1207.3477 [hep-ph]]
27 H. Y. Cheng, C. Y. Cheung, and C. W. Hwang, Phys. Rev. D, **55**: 1559 (1997) doi: 10.1103/PhysRevD.55.1559 [hep-ph/9607332]
28 R. H. Li, C. D. Lü, W. Wang, F. S. Yu and Z. T. Zou, Phys. Lett. B, **767**: 232 (2017) doi: 10.1016/j.physletb.2017.02.003 [arXiv: 1701.03284 [hep-ph]]
29 W. Wang, F. S. Yu, and Z. X. Zhao, Eur. Phys. J. C, **77**(11): 781 (2017) doi: 10.1140/epjc/s10052-017-5360-1 [arXiv: 1707.02834 [hep-ph]]
30 W. Wang, Z. P. Xing, and J. Xu, Eur. Phys. J. C, **77**(11): 800 (2017) doi: 10.1140/epjc/s10052-017-5363-y [arXiv: 1707.06570 [hep-ph]]
31 Y. J. Shi, W. Wang, Y. Xing, and J. Xu, Eur. Phys. J. C, **78**(1): 56 (2018) doi: 10.1140/epjc/s10052-018-5532-7 [arXiv: 1712.03830 [hep-ph]]
32 H. n. Li, C. D. Lu, and F. S. Yu, Phys. Rev. D, **86**: 036012 (2012) doi: 10.1103/PhysRevD.86.036012 [arXiv: 1203.3120 [hep-ph]]
33 J. M. Link et al (FOCUS Collaboration), Phys. Lett. B, **512**: 277 (2001) doi: 10.1016/S0370-2693(01)00590-1 [hep-ex/0102040]
34 J. M. Link et al (FOCUS Collaboration), Phys. Lett. B, **540**: 25 (2002) doi: 10.1016/S0370-2693(02)02103-2 [hep-ex/0206013]
35 M. Ablikim, D. S. Du, and M. Z. Yang, Phys. Lett. B, **536**: 34 (2002) doi: 10.1016/S0370-2693(02)01812-9 [hep-ph/0201168]
36 J. W. Li, M. Z. Yang, and D. S. Du, HEPNP, **27**: 665 (2003) [hep-ph/0206154]
37 S. Fajfer, A. Prapotnik, P. Singer, and J. Zupan, Phys. Rev. D, **68**: 094012 (2003) doi: 10.1103/PhysRevD.68.094012 [hep-ph/0308100]
38 X. Q. Li and B. S. Zou, Phys. Rev. D, **57**: 1518 (1998) doi: 10.1103/PhysRevD.57.1518 [hep-ph/9709508]
39 S. L. Chen, X. H. Guo, X. Q. Li, and G. L. Wang, Commun. Theor. Phys., **40**: 563 (2003) doi: 10.1088/0253-6102/40/5/563 [hep-ph/0208006]
40 M. Ablikim et al (BESIII Collaboration), Phys. Rev. D, **95**(11): 111102 (2017) doi: 10.1103/PhysRevD.95.111102 [arXiv: 1702.05279 [hep-ex]]
41 H. Y. Cheng, C. K. Chua, and A. Soni, Phys. Rev. D, **71**: 014030 (2005) doi: 10.1103/PhysRevD.71.014030 [hep-ph/0409317]
42 T. M. Yan, H. Y. Cheng, C. Y. Cheung, G. L. Lin, Y. C. Lin, and H. L. Yu, Phys. Rev. D, **46**: 1148 (1992); Phys. Rev. D, **55**: 5851 (1997) doi: 10.1103/PhysRevD.46.1148, 10.1103/PhysRevD.55.5851
43 R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio, and G. Nardulli, Phys. Rept., **281**: 145 (1997) doi: 10.1016/S0370-1573(96)00027-0 [hep-ph/9605342]
44 U. G. Meissner, Phys. Rept., **161**: 213 (1988) doi: 10.1016/0370-1573(88)90090-7
45 N. Li and S. L. Zhu, Phys. Rev. D, **86**: 014020 (2012) doi: 10.1103/PhysRevD.86.014020 [arXiv: 1204.3364 [hep-ph]]
46 T. M. Aliev, K. Azizi, and M. Savci, Phys. Lett. B, **696**: 220 (2011) doi: 10.1016/j.physletb.2010.12.027 [arXiv: 1009.3658 [hep-ph]]
47 T. M. Aliev, K. Azizi, and M. Savci, Nucl. Phys. A, **852**: 141 (2011) doi: 10.1016/j.nuclphysa.2011.01.011 [arXiv: 1011.0086 [hep-ph]]
48 A. Khodjamirian, C. Klein, T. Mannel, and Y.-M. Wang, JHEP, **1109**: 106 (2011) doi: 10.1007/JHEP09(2011)106 [arXiv: 1108.2971 [hep-ph]]
49 K. Azizi, Y. Sarac, and H. Sundu, Phys. Rev. D, **90**(11): 114011 (2014) doi: 10.1103/PhysRevD.90.114011 [arXiv: 1410.7548 [hep-ph]]
50 G. L. Yu, Z. G. Wang, and Z. Y. Li, Chin. Phys. C, **41**(8): 083104 (2017) doi: 10.1088/1674-1137/41/8/083104 [arXiv: 1608.03460 [hep-ph]]
51 K. Azizi, Y. Sarac, and H. Sundu, Nucl. Phys. A, **943**: 159 (2015) doi: 10.1016/j.nuclphysa.2015.09.005 [arXiv: 1501.05084 [hep-ph]]
52 A. Ballon-Bayona, G. Krein, and C. Miller, Phys. Rev. D, **96**(1): 014017 (2017) doi: 10.1103/PhysRevD.96.014017 [arXiv: 1702.08417 [hep-ph]]
53 H. Y. Cheng and C. W. Chiang, Phys. Rev. D, **81**: 074021 (2010) doi: 10.1103/PhysRevD.81.074021 [arXiv: 1001.0987 [hep-ph]]
54 H. Y. Cheng and C. W. Chiang, Phys. Rev. D, **81**: 114020 (2010) doi: 10.1103/PhysRevD.81.114020 [arXiv: 1005.1106 [hep-ph]]
55 R. Aaij et al (LHCb Collaboration), Phys. Rev. Lett., **119**(11): 112001 (2017) doi: 10.1103/PhysRevLett.119.112001 [arXiv: 1707.01621 [hep-ex]]