Thermodynamics and weak cosmic censorship conjecture of 4D Gauss-Bonnet-Maxwell black holes via charged particle absorption*

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Abstract: Recently, the non-trivial solutions for 4-dimensional black holes of Einstein-Gauss-Bonnet gravity had been discovered. In this paper, considering a charged particle entering into a 4-dimensional Gauss-Bonnet-Maxwell black hole, we calculate the black hole thermodynamic properties using the Hamilton-Jacobi equation. In the normal phase space, the cosmological constant and Gauss-Bonnet parameter are fixed, the black hole satisfies the first and second laws of thermodynamics, and the weak cosmic censorship conjecture (WCCC) is valid. On the other hand, in the case of extended phase space, the cosmological constant and Gauss-Bonnet parameter are treated as thermodynamic variables. The black hole also satisfies the first law of thermodynamics. However, the increase or decrease in the black hole's entropy depends on some specific conditions. Finally, we observe that the WCCC is violated for the near-extremal black holes in the extended phase space.

Keywords: 4D Gauss-Bonnet-Maxwell black holes, weak cosmic censorship conjecture, black hole thermodynamic properties, extended phase space

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1 Introduction

Studying the thermodynamic properties of black holes helps in understanding quantum gravity. Considering a particle with negative energy entering into spacetime with an ergoregion (e.g. a rotating black hole), the energy of the black hole could be extracted [1]. Later, Christodoulou found that there was an irreducible mass when a black hole absorbed a particle [2-4]. Due to the relationship between the irreducible mass and entropy, the entropy of the black hole corresponds to its area of horizon (or the square of its irreducible mass) [5, 6]. Furthermore, Hawking *et al.* established the four laws of thermodynamics for black holes, which are similar to those in statistical thermodynamics [7]. Based on the contributions noted above, the thermodynamic properties of black holes can be calculated semi-classically without considering the complete quantum gravity. In addition, Hawking found that black holes can have temperatures in curved spacetime [8].

When Maldacena discovered the AdS/CFT correspondence [9], studies on the asymptotic anti-de Sitter (AdS) black holes became very prevalent. Hawking and Page presented a first-order phase transition between the thermal AdS space and the Schwarzschild AdS black hole [10]. In a following work [11], this result was translated into the language of AdS/CFT, which is called a confinement/deconfinement phase transition. More results on the thermodynamics and phase structures of AdS black holes can be found in Refs. [12-16]. Recently, while considering the asymptotical AdS black holes in the extended phase space, an interesting idea has been proposed to explain the cosmological constant in terms of thermodynamic pressure [17, 18]. Next, many thermodynamic studies on various AdS black holes in the extended phase space were presented [19-25].

Moreover, the weak cosmic censorship conjecture (WCCC) states that the singularity is always present in the event horizon [1]. If an observer has normal initial conditions, then the singularity cannot be observed in the future infinity in any real physical process. Wald proposed a method wherein a test particle was thrown into a black hole in order to test the validity of the WCCC in an extremal Kerr-Newman black hole [26]. However, unfortunately, due to electromagnetic or centrifugal repulsion,

the black hole cannot capture a test particle with relatively large charge or angular momentum to overcharge or overspin itself. Moreover, considering the near-extremal charged/rotating black hole, an overcharged/overspinning black hole could be observed via the absorption of a particle [27-29]. However, because of the backreaction and self-force, the WCCC still holds for these black holes [30-35]. It is worth noting that there is no strong evidence to prove the WCCC, though many papers have discussed its validity in various black holes [36-76]. In particular, the Reissner-Nordstrom (RN)-AdS black hole via the charged particle absorption was considered in the normal and extended phase spaces [49, 77]. The first law of thermodynamics and the WCCC were satisfied, meanwhile the second law of thermodynamics was held near the extreme value. It is noteworthy that the second law of thermodynamics is always valid for a RN-AdS black hole in the normal phase space. In Ref. [62], considering charged particle absorption, the authors studied the thermodynamics and WCCC of Gauss-Bonnet AdS Black Holes in higher dimensions ($D > 4$).

On the other hand, it is well known that non-trivial, static, and spherically symmetric solutions only exist in Gauss-Bonnet gravity when the number of spacetime dimensions $D > 4$. Otherwise, when $D = 4$, the Gauss-Bonnet term becomes a topological invariant which does not contribute to the gravitational equations of motion. In a recent work [78], the modified Einstein-Gauss-Bonnet gravity was proposed for obtaining the non-trivial solutions in the limit $D \to 4$. The method was about mimicking the dimensional regularization in quantum field theory and rescaled the coupling parameter α to $\alpha/(D-4)$. As a result, the divergence of the Gauss-Bonnet contribution when $D = 4$ could be canceled by the factor $(D-4)$. Then, the non-trivial four-dimensional black hole solutions with Gauss-Bonnet contribution were obtained. These new black hole solutions opened a new window to study the Gauss-Bonnet effect in the lower dimensional theory. It is worth noting that although the divergence of the variation of the Gauss-Bonnet action was canceled and a brand-new black hole solution was introduced, its thermodynamic properties still had some problems in the limit $D \to 4$. Therefore, the coupling replacement $\alpha \to \alpha/(D-4)$ cannot describe the topologically nontrivial solutions, and thus, its thermodynamic result is preliminary and requires considerable future studies [79]. On the other hand, traditionally, while constructing the 4-dimensional dynamics by dimensional reduction from higher-dimensional theory, the degrees of freedom of the extra dimensions cannot be discarded. Besides, the additional fields must be considered in the 4-dimensional theory [79]. Furthermore, in Ref. [80], to understand how the dimensional regularization discards the dynamics of the extra dimensions, the author embedded the 2-dimensional spacetime into a D-dimensional spacetime to determine the map between the metric tensors and the Einstein equations in 2-dimensional and higher dimensional theories. Next, considering the limit, the dynamical equations from the extra dimensions could be discarded. This limiting procedure requires highly symmetric backgrounds, which lead the embedment to be valid. However, it is still unclear how the D-dimensional dynamical equations converge to the 4-dimensional dynamical equations by taking the limit $D \to 4$. Moreover, it is difficult to determine which dimensions are retained in the final action, because the limit $D \to 4$ is only achieved by replacing the coupling constant $\alpha \to \alpha/(D-4)$ and not by modifying the Riemann tensors. This replacement cancels out the vanishing factor of the Gauss-Bonnet term, and therefore introduces the local dynamics from the Gauss-Bonnet contribution. In addition, the solutions of the Gauss-Bonnet gravity with Maxwell theory in AdS space were given and some thermodynamic properties were discussed in Ref. [81]. Then, a substantial amount of work appeared on the thermodynamic properties of 4-dimensional Gauss-Bonnet black holes. In Ref. [82], the thermodynamics and phase structure in the extended phase space were discussed, where the cosmological constant was seen as the thermodynamic pressure. The critical behaviors were studied in Refs. [83, 84]. Furthermore, the authors studied the phase transition and microstructures for the four-dimensional charged AdS black hole in Ref. [85]. In Ref. [86], 4-dimensional Einstein-Gauss-Bonnet AdS black holes were treated as heat engines. In addition to thermodynamic properties, the stabilities of 4-dimensional Gauss-Bonnet black holes were also discussed in recent works [87-90]. In Refs. [87, 91], quasinormal modes and strong cosmic censorship were considered.

In this paper, based on the developments mentioned above, we study the thermodynamics and WCCC for this new 4-dimensional Gauss-Bonnet-Maxwell black hole. To be specific, we are trying to verify the thermodynamic laws and the WCCC by throwing a test particle into an over-charged 4-dimensional Gauss-Bonnet-Maxwell black hole not only in the normal phase space but also in the extended phase space. In the extended phase space, both the cosmological constant and the Gauss-Bonnet parameter are treated as thermodynamic variables, which leads to some interesting results. For example, in the extended phase space, the black hole thermodynamics in the Gauss-Bonnet gravity with quadratic nonlinear electrodynamics are discussed in Ref. [92] and the thermodynamic properties of Gauss-Bonnet-Born-Infeld-massive black holes are studied in Ref. [93]. On the other hand, in Ref. [94], the authors considered the thermodynamic behavior of the Gauss-Bonnet-massive gravity with the power-Maxwell field in the normal space. In this paper, in the normal phase space, when a charged particle enters into a 4-dimensional Gauss-Bonnet-Maxwell black hole, the first and second laws of thermodynamics are still sat-

isfied. However, the results are different in the extended phase space. Considering the thermodynamics laws in the extended phase space, the first law of thermodynamics is also satisfied, but the second law is indefinite. If we assume the Gauss-Bonnet parameter does not change after the black hole absorbs a charged particle, the second law is violated for extremal and near-extremal black holes. On the other hand, considering the WCCC, there are still some differences between the normal and the extended phase spaces. For near-extremal black holes, the WCCC is still valid in the normal phase space. However for the extended phase space, the WCCC is violated. Furthermore, after absorbing the charged particle, the extremal black hole becomes non-extremal in the normal phase space, but the extremal black hole stays extremal in the extended phase space.

The rest of this paper is organized as follows. In section 2, we derive the general solutions of the 4-dimensional Gauss-Bonnet-Maxwell black holes and their thermodynamic properties. In section 3, we first review the Hamilton-Jacobi equation for a particle entering into the black hole in curved spacetime. Then we obtain the thermodynamics of the black hole in the normal and extended phase space, and discuss the first and second laws of thermodynamics. In section 4, we throw a charged particle into the black hole to test the WCCC in both phase spaces. In section 5, we give conclusions. We assume $G = \hbar = c = k_B = 1$ for simplicity in this paper.

2   4-dimensional Gauss-Bonnet-Maxwell gravity

In this section, we briefly review the Gauss–Bonnet gravity coupled to the Maxwell theory in 4-dimensions. We will also discuss its thermodynamic properties. Based on Ref. [78], the Gauss-Bonnet-Maxwell theory is described by the action

$$S = \frac{1}{16\pi} \int_{\mathcal{M}} \mathrm{d}^D x \sqrt{-g} \Big[R - 2\Lambda + \frac{\alpha}{D-4} \Big(R^2 - 4R_{\mu\nu} R^{\mu\nu} + R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \Big) - F^{\mu\nu} F_{\mu\nu} \Big], \quad (1)$$

where $\Lambda = -\dfrac{(D-1)(D-2)}{2l^2}$ is the cosmological constant, l is the AdS radius, α is the Gauss-Bonnet coupling constant, and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field-strength tensor, in which A_μ is the gauge potential. Varying the action (1) yields the equations of motion,

$$R_{\mu\nu} - \frac{1}{2}(R - 2\Lambda) g_{\mu\nu} + H_{\mu\nu} = 8\pi T_{\mu\nu}, \quad \nabla_\mu F^{\mu\nu} = 0, \quad (2)$$

where

$$
\begin{aligned}
H_{\mu\nu} = & -\frac{1}{2} \frac{\alpha}{D-4} \Big(R^2 - 4R_{\mu\nu} R^{\mu\nu} + R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \Big) g_{\mu\nu} \\
& + 2 \frac{\alpha}{D-4} \Big(R R_{\mu\nu} - 2R_{\mu\alpha} R^{\alpha\beta} g_{\beta\nu} - 2R_{\mu\lambda\nu\sigma} R^{\lambda\sigma} \\
& + g_{\beta\nu} R_{\mu\gamma\sigma\lambda} R^{\beta\gamma\sigma\lambda} \Big), \\
T_{\mu\nu} = & \frac{1}{4\pi} \Big(-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} g_{\mu\nu} + F_\mu^\lambda F_{\nu\lambda} \Big).
\end{aligned} \quad (3)
$$

Considering a 4-dimensional static spherically symmetric black hole ansatz, we take the following metric and vector potential

$$
\begin{aligned}
\mathrm{d}s^2 = & -f(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2 \big(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2 \big), \\
A = & A_t(r)\mathrm{d}t.
\end{aligned} \quad (4)
$$

After setting $D \to 4$, the equations of metric function $f(r)$ and vector potential $A_t(r)$ are written as

$$
\begin{aligned}
0 = & 1 - f(r) - rf'(r) + \frac{3}{l^2}r^2 + + 2\alpha f'(r)(f(r) - 1)r^{-1} \\
& - \alpha (f(r) - 1)^2 r^{-2} + (\partial_r A_t(r))^2 r^2,
\end{aligned} \quad (5)
$$

$$0 = \big[r^2 \partial_r A_t(r) \big]'. \quad (6)$$

By solving Eq. (5) and Eq. (6), one can obtain the solutions for metric function and vector potential [81]

$$f(r) = 1 + \frac{r^2}{2\alpha} \left[1 - \sqrt{1 + 4\alpha \left(-\frac{1}{l^2} + \frac{2M}{r^3} - \frac{Q^2}{r^4} \right)} \right], \quad (7)$$

$$A_t(r) = -\frac{Q}{r}, \quad (8)$$

where M is the ADM mass of the black hole and Q is the black hole charge. The thermodynamic properties of the black hole can be defined on the black hole horizon r_+, which is determined by $f(r_+) = 0$. The Hawking temperature, electrical potential, and entropy of the black hole are given by [81, 85]

$$T \equiv \frac{f'(r_+)}{4\pi} = \frac{-\alpha + r_+^2 + 3\frac{r_+^4}{l^2} - Q^2}{4\pi (r_+^3 + 2\alpha r_+)}, \quad (9)$$

$$\Phi \equiv \int_{r_+}^\infty A_t'(r) = -A_t(r_+), \quad (10)$$

$$S \equiv \int \frac{\mathrm{d}M}{T} = \pi r_+^2 + 4\pi\alpha \ln \frac{r_+}{\sqrt{\alpha}}, \quad (11)$$

where $A_t(r)$ goes to zero at $r = \infty$, the electrostatic potential Φ is a conjugated thermodynamic variable to black hole charge Q. It is worth noting that $\sqrt{\alpha}$ in Eq. (11) is from the identification with an integral constant, and the purpose of this identification is to ensure $\ln \frac{r_+}{\sqrt{\alpha}}$ is dimen-

sionless and the Smarr relation (25) associated withthe entropy is consistent with the higher-dimensional form.

3   Thermodynamics via charged particle absorption

In this section, we study the black hole thermodynamics of the Einstein-Gauss-Bonnet gravity coupled to the Maxwell theory through a charged particle entering the black hole horizon. Due to the conservation of energy and charge, after absorbing a charged particle, the mass and charge of the black hole would change. Furthermore, the other thermodynamic properties of the black hole may also change. The purpose of this section is to check whether the change in the thermodynamic variables of the black hole will violate the first and second laws of thermodynamics in the normal and extended phase spaces.

At first, we briefly review the relationship of the test particle's energy with its radial momentum and potential energy before the particle enters the horizon. The Hamilton-Jacobi equation of the test particle is given by [61]:

$$-\frac{[E + qA_t(r)]^2}{f(r)} + \frac{[P^r(r)]^2}{f(r)} + \frac{L^2}{r^2} = m^2, \quad (12)$$

where L is the particle's angular momentum and $P^r(r)$ is the particle's radial momentum. It is worth mentioning that $P^r(r_+)$ is finite and proportional to the Hawking temperature of the black hole [95, 96]. Since the energy of the particle is required to be a positive value [2, 4], we can rewrite Eq. (12) as

$$E = -qA_t(r) + \sqrt{f(r)\left(m^2 + \frac{L^2}{r^2}\right) + [P^r(r)]^2}. \quad (13)$$

At the horizon $r = r_+$, the above equation reduces to

$$E = q\Phi + \left|P^r(r_+)\right|, \quad (14)$$

which relates the energy of the particle to its radial momentum and potential energy just before the particle enters the horizon.

For convenience, before the subsequent discussions on the thermodynamic properties via charged particle absorption, we present the following formulas:

$$\left.\frac{\partial f(r)}{\partial r}\right|_{r=r_+} = 4\pi T, \quad \left.\frac{\partial f(r)}{\partial M}\right|_{r=r_+} = -\frac{2}{r_+ + \frac{2\alpha}{r_+}},$$

$$\left.\frac{\partial f(r)}{\partial l}\right|_{r=r_+} = -\frac{2r_+^2}{l^3}\frac{1}{1 + \frac{2\alpha}{r_+^2}},$$

$$\left.\frac{\partial f(r)}{\partial Q}\right|_{r=r_+} = \frac{2\Phi}{r_+ + \frac{2\alpha}{r_+}}, \quad \left.\frac{\partial f(r)}{\partial \alpha}\right|_{r=r_+} = \frac{1}{r_+^2 + 2\alpha}. \quad (15)$$

3.1   Normal phase space

In the normal phase space, only the black hole mass

M and black hole charge Q are the thermodynamic variables. We assume that the charged particle, which enters into the horizon of the black hole, has the energy E and charge q. The black hole changes its properties from (M, Q) to $(M + dM, Q + dQ)$ after absorbing a charged particle. In the normal phase space, the black hole mass M is considered as the internal energy U of the black hole. Based on the law of conservation of energy and charge, we have the formulas:

$$dM = E, \quad dQ = q. \quad (16)$$

Before the charged particle enters the black hole horizon, the outer horizon radius r_+ satisfies

$$f(r_+; M, Q) = 0. \quad (17)$$

Then after absorbing the charged particle, the black hole horizon radius is written as $r_+ + dr_+$, which also satisfies

$$f(r_+ + dr_+; M + dM, Q + dQ) = 0. \quad (18)$$

Therefore, the total differential of f can be obtained by

$$\left.\frac{\partial f(r)}{\partial r}\right|_{r=r_+} dr_+ + \left.\frac{\partial f(r)}{\partial M}\right|_{r=r_+} dM + \left.\frac{\partial f(r)}{\partial Q}\right|_{r=r_+} dQ = 0. \quad (19)$$

Substituting Eqs. (15) into Eq. (19), and then combining Eq. (11) to remove the dr_+ term, we can obtain the first law of thermodynamics

$$dM = \Phi dQ + T dS. \quad (20)$$

Furthermore, using Eqs. (14), (16), and (20), the variation of entropy becomes

$$dS = \frac{|P^r(r_+)|}{T} > 0, \quad (21)$$

which shows that absorbing a charged particle in normal phase space does not violate the second law of thermodynamics.

3.2   Extended phase space

In the extended phase space, not only the black hole mass M and the black hole charge Q, but also the cosmological constant l and the Gauss–Bonnet parameter α in metric function $f(r)$ are thermodynamic variables. In this case, we define the thermodynamic pressure of the black hole using the cosmological constant [17, 18]:

$$P \equiv -\frac{\Lambda}{8\pi} = \frac{3}{8\pi l^2}. \quad (22)$$

and the conjugate thermodynamic volume of the black hole is given by

$$V = \left(\frac{\partial M}{\partial P}\right)_{S,Q,\alpha} = \frac{4\pi}{3} r_+^3, \quad (23)$$

where we use Eq. (7) and $f(r_+) = 0$. Furthermore, the conjugate quantity of Gauss-Bonnet parameter α is $\mathcal{A}$, which is defined as [85]:

$$\mathcal{A} = \left(\frac{\partial M}{\partial \alpha}\right)_{S,Q,P} = \frac{1}{2r_+} + 2\pi T\left(1 - 2\ln\frac{r_+}{\sqrt{\alpha}}\right). \quad (24)$$

The Smarr formula hence can be confirmed as

$$M = 2TS + \Phi Q - 2PV + 2\mathcal{A}\alpha, \quad (25)$$

which is consistent with the higher-dimensional form for the Smarr formula [21]. Moreover, in the extended phase space, the black hole mass M should be treated as the enthalpy H instead of internal energy U of the black hole [21]. Therefore, in the extended phase space, when a charged particle of energy E and charge q enters the black hole horizon, it causes the internal energy and charge of the black hole to change as

$$dU = d(M - PV) = E, \quad dQ = q. \quad (26)$$

Irrespective of whether the radius r takes the initial black hole horizon radius r_+ or the changed horizon radius $r_+ + dr_+$ after absorbing a charged particle, the metric

function $f(r)$ must be set to zero. Moreover, we can obtain the infinitesimal changes in M, Q, l, α, and r_+:

$$\frac{\partial f(r)}{\partial r}\Big|_{r=r_+} dr_+ + \frac{\partial f(r)}{\partial M}\Big|_{r=r_+} dM + \frac{\partial f(r)}{\partial Q}\Big|_{r=r_+} dQ$$
$$+ \frac{\partial f(r)}{\partial l}\Big|_{r=r_+} dl + \frac{\partial f(r)}{\partial \alpha}\Big|_{r=r_+} d\alpha = 0. \quad (27)$$

Substituting Eq. (15) into Eq. (27), we can easily obtain the first law of thermodynamics of a Gauss-Bonnet-Maxwell black hole in the extended phase space,

$$dM = \Phi dQ + T dS + V dP + \mathcal{A} d\alpha, \quad (28)$$

where dl is replaced with dP through Eq. (22). Combining Eq. (26) and the first law of thermodynamics (27), we obtain that

$$|P^r(r_+)| = T dS - P dV + \mathcal{A} d\alpha. \quad (29)$$

Using Eqs. (22)-(24) and substituting Eq. (11) back into Eq. (29), we can obtain the change in the black hole entropy:

$$dS = \frac{\left(1 + \frac{2\alpha}{r_+^2}\right)|P^r(r_+)| - \left\{\frac{1}{2r_+}\left(1 + \frac{2\alpha}{r_+^2}\right) + \left[\left(1 + \frac{2\alpha}{r_+^2}\right)T - \frac{3r_+}{4\pi l^2}\right]2\pi\left(1 - 2\ln\frac{r_+}{\sqrt{\alpha}}\right)\right\}d\alpha}{\left(1 + \frac{2\alpha}{r_+^2}\right)T - \frac{3r_+}{4\pi l^2}}. \quad (30)$$

Based on Eq. (9), for a large enough T, the denominator in Eq. (30) becomes

$$\left(1 + \frac{2\alpha}{r_+^2}\right)T - \frac{3r_+}{4\pi l^2} > 0. \quad (31)$$

Otherwise, for a relatively small T, the denominator is negative. Since $d\alpha$ is arbitrary, the sign of the numerator in Eq. (30) is indefinite. In the extended phase space, the entropy can increase or decrease depending on the value of $d\alpha$. Considering the "restricted" extended phase space with $d\alpha = 0$, the change in the black hole entropy becomes

$$dS = \frac{4\pi r_+^3\left(1 + \frac{2\alpha}{r_+^2}\right)|P^r(r_+)|}{\left(1 + \frac{2\alpha}{r_+^2}\right)T - \frac{3r_+}{4\pi l^2}}, \quad (32)$$

which shows that the second law of thermodynamics is not satisfied for the extremal or near-extremal black hole. When the black hole is far enough from extremality, the second law is satisfied in the "restricted" extended phase space.

4   Weak cosmic censorship conjecture

In this section, we will check the validity of the WCCC when a charged particle enters into the black hole horizon. We assume that the initial black hole is extremal or near extremal before absorbing a charged particle. Since the test particle has a very small energy and charge com-

pared to the black hole, to become a naked singularity requires the black hole to be close to the extremality. Therefore, here we assume that the initial Gauss-Bonnet-Maxwell black hole is near-extremal, which possesses two horizons. Between these two horizons, there exists one and only one minimum point at $r = r_{\min}$ for $f(r)$. Moreover, the minimum value of $f(r)$ is not greater than zero,

$$\delta \equiv f(r_{\min}) \leqslant 0, \quad (33)$$

where $\delta = 0$ corresponds to the extremal black hole. After the black hole absorbs a charged particle, the minimum point moves to $r_{\min} + dr_{\min}$. For the final black hole solution, if the minimum value of $f(r_{\min} + dr_{\min})$ is still not greater than zero, then there is an event horizon. Otherwise, the final black hole solution is over the extremal limit, and the WCCC is violated. Again, we first present some useful formulas:

$$\frac{\partial f(r)}{\partial r}\Big|_{r=r_{\min}} = 0,$$

$$\frac{\partial f(r)}{\partial M}\Big|_{r=r_{\min}} = -\frac{2}{r_{\min} + \frac{2\alpha}{r_{\min}}(1-\delta)},$$

$$\frac{\partial f(r)}{\partial l}\Big|_{r=r_{\min}} = -\frac{2r_{\min}^2}{l^3}\frac{1}{1+\frac{2\alpha}{r_{\min}^2}(1-\delta)},$$

$$\frac{\partial f(r)}{\partial Q}\Big|_{r=r_{\min}} = \frac{2(\Phi + A_t(r_+) - A_t(r_{\min}))}{r_{\min} + \frac{2\alpha}{r_{\min}}(1-\delta)},$$

$$\frac{\partial f}{\partial \alpha}\Big|_{r=r_{\min}} = \frac{(1-f(r_{\min}))^2}{r_{\min}^2 + 2\alpha(1-\delta)}. \tag{34}$$

4.1   Normal phase space

In the normal phase space, the charged particle with the energy E and charge q enters into the black hole, which makes the black hole shift from the initial state (M, Q) to the final state $(M + dM, Q + dQ)$, where dM and dQ are given in Eq. (16). Moreover, the minimum value of $f(r)$ moves from $f(r_{\min})$ to $f(r_{\min} + dr_{\min})$, where the final state $f(r_{\min} + dr_{\min})$ can be rewritten in terms of the initial state δ:

$$f(r_{\min} + dr_{\min}; M + dM, Q + dQ)$$

$$= \delta + \frac{\partial f}{\partial r}\Big|_{r=r_{\min}} dr_{\min} + \frac{\partial f}{\partial M}\Big|_{r=r_{\min}} dM + \frac{\partial f}{\partial Q}\Big|_{r=r_{\min}} dQ$$

$$= \delta - \frac{2|P^r(r_+)|}{r_{\min} + \frac{2\alpha}{r_{\min}}(1-\delta)} + \frac{2q[A_t(r_+) - A_t(r_{\min})]}{r_{\min} + \frac{2\alpha}{r_{\min}}(1-\delta)}. \tag{35}$$

The extremal black hole implies $r_{\min} = r_+$ and $\delta = 0$. Therefore, the minimum value of the final state metric

function $f(r)$ becomes

$$f(r_{\min} + dr_{\min}) = -\frac{2|P^r(r_+)|}{r_{\min} + \frac{2\alpha}{r_{\min}}} < 0. \tag{36}$$

That is, the extremal black hole becomes non-extremal, after absorbing a charged particle. Furthermore, if the initial black hole is near-extremal, we define ϵ such that

$$r_{\min} = r_+(1-\epsilon), \tag{37}$$

where $0 < \epsilon \ll 1$. Thus, δ is suppressed by ϵ in the near-extremal limit. Moreover, based on (8), the second term in the third line of Eq. (35) can be rewritten as

$$\frac{2q[A_t(r_+) - A_t(r_{\min})]}{r_{\min} + \frac{2\alpha}{r_{\min}}(1-\delta)} = \frac{2Qq\epsilon}{r_+^2(1-\epsilon)^2 + 2\alpha(1-\delta)}. \tag{38}$$

Therefore, in the near-extremal black hole, considering the test particle limit, the third term of Eq. (35) can be neglected. Then Eq. (35) becomes

$$f(r_{\min} + dr_{\min}) = \delta - \frac{2|P^r(r_+)|}{r_{\min} + \frac{2\alpha}{r_{\min}}(1-\delta)} < 0, \tag{39}$$

which implies that the near-extremal black hole is still non-extremal after the absorption. As a result, in the normal phase space, the WCCC is satisfied for the extremal and near-extremal Gauss-Bonnet-Maxwell black holes upon the absorption of a charged particle.

4.2   Extended phase space

In this case, absorbing a charged particle makes the parameters of the black hole change from (M, Q, l, α) to $(M + dM, Q + dQ, l + dl, \alpha + d\alpha)$, and $r_{\min}$ changes to $r_{\min} + dr_{\min}$. For the final state at $r = r_{\min} + dr_{\min}$, the minimum value of $f(r)$ is

$$f(r_{\min} + dr_{\min}, M + dM, Q + dQ, \alpha + d\alpha) = \delta + \frac{\partial f}{\partial M}\Big|_{r=r_{\min}} dM + \frac{\partial f}{\partial Q}\Big|_{r=r_{\min}} dQ + \frac{\partial f}{\partial l}\Big|_{r=r_{\min}} dl + \frac{\partial f}{\partial \alpha}\Big|_{r=r_{\min}} d\alpha$$

$$= \delta + (-TdS + q[A_t(r_+) - A_t(r_{\min})])\frac{2}{r_{\min} + \frac{2\alpha}{r_{\min}}(1-\delta)} + \frac{r_{\min}^3}{l^3}\frac{2}{r_{\min} + \frac{2\alpha}{r_{\min}}(1-\delta)}\left[\frac{r_+^3}{r_{\min}^3} - 1\right]dl$$

$$+ \frac{1}{r_{\min}^2 + 2\alpha(1-\delta)}\left[(1-\delta)^2 - \left(\frac{r_{\min}}{r_+} + 4\pi T r_{\min}\left(1 - 2\ln\frac{r_+}{\sqrt{\alpha}}\right)\right)\right]d\alpha, \tag{40}$$

where we use Eq. (34) for the derivatives of $f(r)$. If the initial black hole is extremal, we have $r_{\min} = r_+$, $T = 0$, and $\delta = 0$. Thus, we find that the minimum value of $f(r)$ of the final black hole becomes

$$f(r_{\min} + dr_{\min}) = 0, \tag{41}$$

which means that the extremal black hole is still extremal after absorbing the test particle. On the other hand, considering the near-extremal black hole, we define an infinitesimal quantity ϵ as (37). Then substituting Eq. (30) into (40), we can get

$$f(r_{\min} + \mathrm{d}r_{\min}) = \delta - \frac{2}{r_+(1-\epsilon) + \dfrac{2\alpha}{r_+(1-\epsilon)}(1-\delta)} T \frac{\left(1 + \dfrac{2\alpha}{r_+^2}\right)|P^r(r_+)|}{\left(1 + \dfrac{2\alpha}{r_+^2}\right)T - \dfrac{3r_+}{4\pi l^2}} + \frac{1}{r_+^2(1-\epsilon)^2 + 2\alpha(1-\delta)}\left[\frac{\dfrac{3r_+}{4\pi l^2}}{\left(1 + \dfrac{2\alpha}{r_+^2}\right)T - \dfrac{3r_+}{4\pi l^2}} + (1-\delta)^2\right]\mathrm{d}\alpha$$

$$+ \frac{\epsilon}{r_+^2(1-\epsilon)^2 + 2\alpha(1-\delta)}\left[2Qq + 2(1-\epsilon)\left(\epsilon^2 - 3\epsilon + 3\right)\frac{r_+^4}{l^3}\mathrm{d}l - \frac{\dfrac{3r_+}{4\pi l^2}}{\left(1 + \dfrac{2\alpha}{r_+^2}\right)T - \dfrac{3r_+}{4\pi l^2}}\epsilon\,\mathrm{d}\alpha\right].$$

$$(42)$$

Since the quantity ϵ is infinitesimal, the term in the fourth line of Eq. (42) can be neglected. However, as we discussed in the final part of section 3, the temperature in the near-extremal black hole is low enough for the denominator in the second term of the second line of Eq. (42) to be neglected. Therefore, this term is positive. Moreover, $\mathrm{d}\alpha$ is arbitrary; hence, the sign of the third line in Eq. (42) is indefinite. In general, the test particle can overcharge the near-extremal Gauss-Bonnet black hole in 4-dimensions, which invalidates the WCCC.

5   Conclusion

In this paper, we first reviewed the solutions of the 4-dimensional Gauss-Bonnet-Maxwell black holes. Then, we obtained the thermodynamic quantities of the black hole and examined the first and second laws of thermodynamics by throwing a charge practice into the black hole. Finally, we verified the WCCC for a Gauss-Bonnet black hole coupled to Maxwell theory in the normal phase space and extended phase space. Our results are summarized as follows (Table 1):

Table 1.   Results for the first and second laws of thermodynamics and the WCCC, which were tested for a GaussBonnet-Maxwell black hole by the absorption of a charged particle in the test particle limit.

	Normal Phase Space	Extended Phase Space
1st law	Satisfied.	Satisfied.
2nd law	Satisfied.	Indefinite. If $\mathrm{d}\alpha = 0$ is imposed, the 2nd law is violated for the extremal and near-extremal black holes.
WCCC	Satisfied for the extremal and near-extremal black holes. After the charged particle absorption, the extremal black hole becomes non-extremal.	Satisfied for the extremal black holes. After the charged particle absorption, the extremal black hole stays extremal. Violated for near-extremal black holes.

As shown in Table 1, after the absorption of a charged particle, the first law of thermodynamics of the 4-dimensional Gauss-Bonnet-Maxwell black hole is still satisfied both in the normal phase space and in the extended phase space. However, the second law of thermodynamics is different in these two cases. In the normal phase space, the second law of thermodynamics is still satisfied. Nevertheless, the second law is indefinite in the extended phase space. More specifically, if we assume the Gauss-Bonnet parameter does not change after the black hole absorbs a charged particle, the second law is violated for the extremal and near-extremal black holes. Furthermore, the WCCC is considered in the normal phase space. When a charged particle enters into a near-extremal black hole, the WCCC is still valid. Meanwhile, for the extremal black hole, the WCCC is violated, and the black hole becomes non-extremal. On the other hand, for the black hole in the extended phase space, if a near-extremal black hole absorbs a charged particle, the WCCC

is still valid. However, for the extremal black hole, the WCCC is violated.

In the near future, it is reasonable to study the extended phase space thermodynamics for 4-dimensional Gauss-Bonnet-Maxwell black holes in a cavity. This study motivates further discussion of the deep relations between the thermodynamic properties of novel 4-dimensional black holes and their boundary conditions. Then, based on the works of [76], it is natural to discuss the validity of thermodynamic laws and the WCCC for 4-dimensional Gauss-Bonnet-Maxwell black holes in a cavity.

Note: The authors considered the same problem only in the normal phase space which appeared on arXiv on April 19, 2020 [97].

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