Spectroscopic study of strangeness = –3 Ω^- baryon*

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Abstract: Ω^- baryon with sss quarks has been investigated through many theoretical studies so far but scarcely observed in experiments. Here, an attempt has been made to explore properties of Ω with hypercentral Constituent Quark Model (hCQM) with a linear confining term. The resonance mass spectra have been obtained for 1*S*–4*S*, 1*P*–4*P*, 1*D*–3*D,* and 1*F*–2*F*. The Regge trajectory has been investigated for the linear nature based on calculated data along with the magnetic moment. The present work has been compared with various approaches and known experimental findings.

Keywords: mass spectra, strange baryon, Regge trajectory, magnetic moment

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I. INTRODUCTION

The discovery of Ω baryon dates back to 1964; nevertheless, it continues to be the least observed in experiments worldwide. Ω^- holds a place in the decuplet family with isospin I = 0 and strangeness S = –3 with sss quarks. The search for missing resonances is the prime aim of hadron spectroscopy to understand the internal dynamics of the quarks inside the system ranging from light to heavy as well as exotic hadrons as depicted by few recent reviews [1, 2]. The motivation behind the current study is to exploit all resonance masses with possible spin-parity assignment. This is the extension of the previous study for non-strange [3, 4] and strange baryons with $S = -1, -2$ [5].

Ω baryon purely belongs to the decuplet representation in a similar manner to Δ, whereas, in principle, Σ and Ξ can be realized as a mixture of octet and decuplet states. In terms of multi-strangeness, Ω baryon is similar to Ξ as both are not easily observed in experiments and no further information has been readily available since the publishing of bubble chamber data. In a recent study of strong decays in the constituent quark model with relativisitc corrections, it was highlighted that, unlike other light baryons, Ω with only strange quarks may be a tool to reach the valence quarks in the baryons when other kaon cloud effects are somehow excluded [6]. Pervin *et al.* [7] have vividly described that multi-strange baryons are produced only as a part of the final state with a very small production cross-section, which complicates the analysis for their study. Recent studies at Belle experiments have provided some results regarding $\Omega(2012)$ through e^+e^- annihilations and into $\Xi^0 K^-$ as well as $\Xi^-\bar{K}^0$decay channels [8]. Earlier, BaBar collaboration attempted to study the spin of $\Omega^-(1672)$ for $J = 3/2$ through processes like $\Omega_c^0 \to \Omega^- K^+$ and $\Xi_c^0 \to \Omega^-\pi^+$[9]. All these observations pose a challenge regarding the underlying mystery of multi strange baryons, especially for S=–3. The upcoming experimental facility at FAIR, $\bar{P}$ANDA-GSI is expected to perform a dedicated study of hyperons, especially at low energy regime [10]. Moreover, a part of BESIII experiment shall be including the strange quark systems [11] and J-PARC facility [12].

The three star state $\Omega(2012)$ has been a puzzling one appearing in Table 1 as the second known state. The discovery of $\Omega(2012)$ by the Belle collaboration sparked a lot of theoretical work on the issue, with pictures inspired by quark models as well as molecular pictures based on the meson-baryon interaction. In various quark models, the masses of the first orbital excitations of states were deemed to $\Omega(2012)$. A recent study has proposed this state to be a molecular one. This state is slightly below $\Xi(1530)\{\bar{K}\}$ threshold so that the binding mechanism could be a coupled channel dynamics [13]. Its characteristic signature could be a three body channel $\Xi\{\bar{K}\}\pi$. Some studies argue that the present information is not

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Table 1. PDG Ω baryon [14].

State	J^P	Status
Ω(1672)	$3/2^+$	****
Ω(2012)	$?^-$	***
Ω(2250)		***
Ω(2380)		**
Ω(2470)		**

sufficient for considering it as a molecular state [15, 16], whereas others disapprove the proposed state [17]. Moreover, a study has been revisited to check the compatibility of molecular picture of 2012 within the coupled channel unitary approach [18]. Xiao *et al.* have studied the strong decays within chiral quark model to understand the structure of the Ω(2012) state [19]. There are several models to study the Ω baryon properties theoretically and phenomenologically such as quark pair-creation [20], QCD Sum rule [21], Glozman-Riska model [22], algebraic model by Bijker [23], and large-N_c analysis [24, 25]. Recently, A. Arifi *et al.* have investigated the decay properties of Ξ and Ω baryons including Roper-like resonances using relativistic corrections in the constituent quark model [6]. Our group has also attempted to explore light, strange baryon through Regge Phenomenology [26]. The Nambu-Jona-Lasinio (NJL) approach has been used for calculating the mixing of three and five components in low-lying Ω states with negative parity [27]. The partial wave analysis of light baryons is also a very important tool for the spectroscopy of narrow experimental states [28–31]. The details of these models through comparison are described in Section III.

In the present article, we study the resonance mass spectra of Ω^- baryon through a non-relativistic model. Section II describes the potential terms used to obtain the resonance mass with spin-dependent and correction parts. Section III sketches the results obtained through the model and summarizes the comparison with other models. Sections IV and V exploit the Regge trajectories and magnetic moment properties leading to the conclusion of the study.

II. THEORETICAL BACKGROUND

The present study is based on the hypercentral Constituent Quark Model (hCQM), i.e., a non-relativistic approach [32, 33]. The baryons are composed of three quarks confined within and interacting by a potential that is considered to be hypercentral. The hyperspherical coordinates are given by the angles Ω_ρ, Ω_λ along with hyperradius x and hyperangle ξ, which are written in terms of Jacobi coordinates as in Fig. 1 [34, 35],

$$\boldsymbol{\rho} = \frac{1}{\sqrt{2}}(\boldsymbol{r}_1 - \boldsymbol{r}_2), \tag{1a}$$

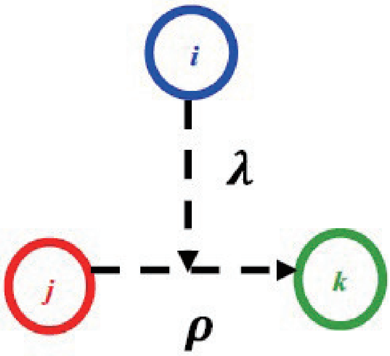

Fig. 1. (color online) Representation of the three-body system [36].

$$\boldsymbol{\lambda} = \frac{(m_1\boldsymbol{r}_1 + m_2\boldsymbol{r}_2 - (m_1+m_2)\boldsymbol{r}_3)}{\sqrt{m_1^2+m_2^2+(m_1+m_2)^2}}, \tag{1b}$$

where r_i and m_i correspond to the internal distance between given two quarks and their masses, respectively.

The hyperradius and hyperangle are defined as

$$x = \sqrt{\rho^2+\lambda^2}, \quad \xi = \arctan\left(\frac{\rho}{\lambda}\right). \tag{2}$$

The hyperradius x is a one-dimensional coordinate, which encloses, simultaneously, the effects of the three-body interaction. The quarks are pictured as connected by gluonic strings where the potential increases linearly with the radius x. The reduced masses with Jacobi co-ordinates $\boldsymbol{\rho}$ and $\boldsymbol{\lambda}$ are given by

$$m_\rho = \frac{2m_1m_2}{m_1+m_2}, \quad m_\lambda = \frac{2m_3(m_1^2+m_2^2+m_1m_2)}{(m_1+m_2)(m_1+m_2+m_3)}. \tag{3}$$

The kinetic energy operator in the center of mass frame is written as

$$-\frac{\hbar^2}{2m}(\Delta_\rho+\Delta_\lambda) = \frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{5}{x}\frac{\partial}{\partial x}+\frac{L^2(\Omega)}{x^2}\right). \tag{4}$$

Here, $L^2(\Omega) = L^2(\Omega_\rho,\Omega_\lambda,\xi)$ is the quadratic Casimir operator for the six-dimensional rotational group whose eigenfucntions are hyperspherical harmonics satisfying

$$L^2(\Omega_\rho,\Omega_\lambda,\xi)Y_{[\gamma]l_\rho l_\lambda}(\Omega_\rho,\Omega_\lambda,\xi) = -\gamma(\gamma+4)Y_{[\gamma]l_\rho l_\lambda}(\Omega_\rho,\Omega_\lambda,\xi), \tag{5}$$

$\gamma = 2n+l_\rho+l_\lambda$ is the grand angular quantum number. Thus, the hyper-radial part of the wave-function, as determined by hypercentral Schrodinger equation, is

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}x^2}+\frac{5}{x}\frac{\mathrm{d}}{\mathrm{d}x}-\frac{\gamma(\gamma+4)}{x^2}\right]\psi(x)=-2m[E-V(x)]\psi(x). \quad (6)$$

Here $l(l+1)\to 15/4+\gamma(\gamma+4)$. The hyperradial wavefunction $\psi(x)$ is completely symmetric for exchange of the quark coordinates using the orthogonal basis [37]. The expansion of the quark interaction term is written as

$$\sum_{i<j} V(r_{ij})=V(x)+... \quad (7)$$

The potential with the first term gives the hypercentral approximation, which has three-body character as not a single pair of coordinates can be disentangled from the third one. The Hamiltonian of the system is written with a potential term solely dependent on the hyperradius x of the three body system.

$$H=\frac{P^2}{2m}+V^0(x)+V_{SD}(x), \quad (8)$$

where $m=\dfrac{2m_\rho m_\lambda}{m_\rho+m_\lambda}$ is the reduced mass. The potential is solely hyperradius dependent. So, it consists of a Coulomb-like term and a linear term acting as confining part.

$$V^0(x)=-\frac{\tau}{x}+\alpha x. \quad (9)$$

Here, $\tau=(2/3)\alpha_s$ with $\alpha_s$representing the running coupling constant.

$$\alpha_s=\frac{\alpha_s(\mu_0)}{1+\left(\dfrac{33-2n_f}{12\pi}\right)\alpha_s(\mu_0)\ln\left(\dfrac{m_1+m_2+m_3}{\mu_0}\right)}. \quad (10)$$

Here, α_s is 0.6 at $\mu_0=1$ GeV and n_f is the number of active quark flavors whose value here is 3, whereas α is the string tension of the confinement part of the potential. Moreover, α is state dependent and is obtained by fixing the value using the experimental ground state mass of the baryon [38, 39]. The constituent quark mass is considered as m_s = 0.500 GeV. The model parameters used for the ground state are as shown in Table 2.

If considering the chiral quark model, the low-energy regime shall be well established as the spontaneously broken $SU(3)$ chiral symmetry scale is different from that of the QCD confinement scale. For three body higher excited states, the relative position of positive and negative parity states can be fixed by the interplay of relativistic kinematics and pion exchange interaction, playing the role of one-gluon exchange potential. Thus, the higher terms in Goldstone exchange will allow us to incorporate the hyperfine, and spin-singlet and triplet splitting [40–42].

Table 2. Ground state model parameters.

m_s/GeV	α_s	α/GeV^2
0.500	0.5109	0.0129

The $V_{SD}(x)$ is added for incorporating spin-dependent contributions through $V_{SS}(x)$, $V_{\gamma S}(x)$ and $V_T(x)$as spin-spin, spin-orbit, and tensor terms, respectively. These interactions arise due to v^2/c^2 effects in non-relativistic expansion and by the standard Breit-Fermi expansion as described by Voloshin [43].

$$V_{SD}(x)=V_{SS}(x)(\boldsymbol{S}_\rho\cdot\boldsymbol{S}_\lambda)+V_{\gamma S}(x)(\boldsymbol{\gamma}\cdot\boldsymbol{S}) +V_T\left[S^2-\frac{3(\boldsymbol{S}\cdot\boldsymbol{x})(\boldsymbol{S}\cdot\boldsymbol{x})}{x^2}\right], \quad (11)$$

$$V_{SS}(x)=\frac{1}{3m_\rho m_\lambda}\nabla^2 V_V, \quad (12)$$

$$V_{\gamma S}(x)=\frac{1}{2m_\rho m_\lambda x}\left(3\frac{\mathrm{d}V_V}{\mathrm{d}x}-\frac{\mathrm{d}V_S}{\mathrm{d}x}\right), \quad (13)$$

$$V_T(x)=\frac{1}{6m_\rho m_\lambda}\left(3\frac{\mathrm{d}^2V_V}{\mathrm{d}x^2}-\frac{1}{x}\frac{\mathrm{d}V_V}{\mathrm{d}x}\right), \quad (14)$$

where $V_V=\tau/x$ and $V_S=\alpha x$ are the vector and scalar part of the potential. However, instead of the spin-spin interaction presented by delta function, we have employed a smear function of the form, which is detailed in previous works [33, 44, 45]. Further, $S=S_\rho+S_\lambda$, where S_ρ and S_λ are the spin vectors associated with the $\boldsymbol{\rho}$ and $\boldsymbol{\lambda}$ variables, respectively.

$$V_{SS}(x)=\frac{-A}{6m_\rho m_\lambda}\frac{\mathrm{e}^{-x/x_0}}{xx_0^2}. \quad (15)$$

Here, x_0 is the hyperfine parameter, with value $x_0=1$ and A is a state dependent parameter consisting of an arbitrary constant. The form of A is chosen as $A=A_0/\sqrt{n+l+\frac{1}{2}}$, wherein the value of $A_0=28$ for determining the ground state value $(1S(3/2)^+$ 1672 MeV) as well as other radially excited states. Similarly, the other parameters are determined for obtaining the experimentally known ground state mass, i.e., 1672 MeV, in the case of Ω. In addition, masses with first order correction as $\dfrac{1}{m}V^1(x)$ are taken into account through

$$V^1(x)=-C_FC_A\frac{\alpha_s^2}{4x^2}, \quad (16)$$

where $C_F = \dfrac{2}{3}$ and $C_A = 3$ are Casimir elements of the fundamental and adjoint representation.

Numerical solutions of Schrodinger equation has been obtained using Mathematica notebook [46].

III. RESULTS AND DISCUSSION FOR THE RESONANCE MASS SPECTRA

In the present work, $1S$–$4S$, $1P$–$4P$, $1D$–$3D$, and $1F$–$2F$ states have been obtained for $S=1/2$ and $S=3/2$ spin configurations with all possible J^P values in Tables 3–6. Moreover, $Mass_{\rm cal}1$ and $Mass_{\rm cal}2$ correspond to the resonance masses without and with the first order correction term, respectively. Tables 7 and 8 show comparisons of the obtained results with various approaches for positive and negative parity states. The ground state $\Omega(1672)$ is nearly the same for many approaches with a variation of 20–30 MeV in few cases depending on the approach.

Faustov *et al.* [47] have employed a relativistic quark model approach considering a quark-diquark system. The lower excited states are found to be in strong accordance, whereas for higher excitations, an exact comparison is not

Table 3. Resonance masses of S-state $1S$–$4S$ without and with first order correction to the potential (in MeV).

State	J^P	$Mass_{\rm cal}1$	$Mass_{\rm cal}2$
$1S$	$3/2^+$	1672	1672
$2S$	$3/2^+$	2057	2068
$3S$	$3/2^+$	2429	2449
$4S$	$3/2^+$	2852	2885

Table 4. Resonance masses of P-state $1P$–$4P$ without and with first order correction to the potential (in MeV).

State	J^P	$Mass_{\rm cal}1$	$Mass_{\rm cal}2$
$1^2P_{1/2}$	$1/2^-$	1987	1996
$1^2P_{3/2}$	$3/2^-$	1978	1985
$1^4P_{1/2}$	$1/2^-$	1992	2001
$1^4P_{3/2}$	$3/2^-$	1983	1991
$1^4P_{5/2}$	$5/2^-$	1970	1997
$2^2P_{1/2}$	$1/2^-$	2345	2363
$2^2P_{3/2}$	$3/2^-$	2332	2349
$2^4P_{1/2}$	$1/2^-$	2352	2370
$2^4P_{3/2}$	$3/2^-$	2339	2356
$2^4P_{5/2}$	$5/2^-$	2321	2338
$3^2P_{1/2}$	$1/2^-$	2758	2788
$3^2P_{3/2}$	$3/2^-$	2740	2770
$3^4P_{1/2}$	$1/2^-$	2767	2797
$3^4P_{3/2}$	$3/2^-$	2749	2779
$3^4P_{5/2}$	$5/2^-$	2726	2755
$4^2P_{1/2}$	$1/2^-$	3218	3264
$4^2P_{3/2}$	$3/2^-$	3196	3240
$4^4P_{1/2}$	$1/2^-$	3229	3276
$4^4P_{3/2}$	$3/2^-$	3207	3252
$4^4P_{5/2}$	$5/2^-$	3178	3221

Table 5. Resonance masses of D-state $1D$–$3D$ without and with first order correction to the potential (in MeV).

State	J^P	$Mass_{\rm cal}1$	$Mass_{\rm cal}2$
$1^2D_{3/2}$	$3/2^+$	2269	2288
$1^2D_{5/2}$	$5/2^+$	2250	2267
$1^4D_{1/2}$	$1/2^+$	2291	2311
$1^4D_{3/2}$	$3/2^+$	2276	2295
$1^4D_{5/2}$	$5/2^+$	2257	2275
$1^4D_{7/2}$	$7/2^+$	2233	2249
$2^2D_{3/2}$	$3/2^+$	2671	2703
$2^2D_{5/2}$	$5/2^+$	2646	2676
$2^4D_{1/2}$	$1/2^+$	2699	2733
$2^4D_{3/2}$	$3/2^+$	2681	2713
$2^4D_{5/2}$	$5/2^+$	2656	2686
$2^4D_{7/2}$	$7/2^+$	2623	2652
$3^2D_{3/2}$	$3/2^+$	3122	3166
$3^2D_{5/2}$	$5/2^+$	3092	3135
$3^4D_{1/2}$	$1/2^+$	3157	3201
$3^4D_{3/2}$	$3/2^+$	3134	3178
$3^4D_{5/2}$	$5/2^+$	3103	3146
$3^4D_{7/2}$	$7/2^+$	3065	3107

Table 6. Resonance masses of F-state $1F$–$2F$ without and with first order correction to the potential (in MeV).

State	J^P	$Mass_{\rm cal}1$	$Mass_{\rm cal}2$
$1^2F_{5/2}$	$5/2^-$	2585	2614
$1^2F_{7/2}$	$7/2^-$	2552	2579
$1^4F_{3/2}$	$3/2^-$	2622	2653
$1^4F_{5/2}$	$5/2^-$	2595	2625
$1^4F_{7/2}$	$7/2^-$	2562	2590
$1^4F_{9/2}$	$9/2^-$	2521	2548
$2^2F_{5/2}$	$5/2^-$	3027	3069
$2^2F_{7/2}$	$7/2^-$	2986	3027
$2^4F_{3/2}$	$3/2^-$	3072	3115
$2^4F_{5/2}$	$5/2^-$	3039	3081
$2^4F_{7/2}$	$7/2^-$	2998	3040
$2^4F_{9/2}$	$9/2^-$	2949	2999

possible. In ref [48], the authors studied the spectrum through hyperfine interactions due to two-gluon exchange. For the available states, our current results are very close to those reported, within 50 MeV. Another non-relativistic constituent quark model approach has been utilized by [49]. Y. Oh [50] has investigated the Ω spectrum using Skyrme model. Refs [51] and [52] have exploited the quark model based on chromodynamics, with which some of the present states are also in accordance. E. Klempt [53] has reproduced the few known states through a new baryon mass formula. U. Löring *et al.* have studied the whole light spectrum within a relativistic covariant quark model based on Bethe-Salpeter equation [54]. The BGR collaboration [55] results are based on chirally improved (CI) quarks. For higher J^P values, few approaches are available for comparison.

One puzzling issue remains regarding the Ω(2012) state, i.e. our results vary by 30 MeV and do not exactly reproduce previous measurements. Moreover, this study is not able to precisely comment on the proposed molecular nature of this state. So, the future experimental results would serve as a key towards its understanding.

The results described in Tables 3–6 have been summarized in the increasing order for each J^P value including positive parity in Table 7 and negative parity in Table 8. All the mentioned models appearing in the table for comparison are not sufficient to segregate each state based on the J^P value. The ground state Ω(1672) with $J^P = 3/2^+$ is close to those in [47–49] and [52]. The first state with $J^P = 1/2^+$ is nearly comparable to that in the relativistic approach by Faustov *et al.*

The first excited states with $J^P = 5/2^+$ and $J^P = 7/2^+$ are not very far from most of the comparison. As in the case on negative parity states $J^P = 3/2^-$, [48], [49], [51] and [52], the results are close to 2012 MeV, whereas the present study could not identify exactly the proposed Ω(2012) state. The state $J^P = 7/2^-$ is very close to the results from [47] and [54].

Here, we attempt to assign a tentative spin-parity to the three and two starred states. The Ω(2250) with an experimental mass at 2252 ± 9 is quite comparable to our 1D with $J^P = 5/2^+$. The fourth state Ω(2380) may possibly be a member of the 2P family with $J^P = 1/2^-$ matching our value at 2370 MeV. The last state of Ω(2470) with a mass as 2474 ± 12 might be assigned a 3S state with $J^P = 3/2^+$ as the calculated state 2429 or 2449 MeV.

Table 7. Comparison of present masses with other approaches based on J^P value with positive parity described in the increasing order for all possible spin-parity assignment (in MeV).

J^P	$Mass_{cal}1$	$Mass_{cal}2$	[47]	[48]	[49]	[7]	[50]	[51]	[52]	[54]	[55]
$1/2^+$	2291	2311	2301	2182	2232	2175	2140	2220	2190	2232	2350(63)
	2699	2733		2202		2191		2255	2210	2256	2481(51)
	3157	3201									
$3/2^+$	1672	1672	1678	1673	1672	1656	1694	1635	1675		1642(17)
	2057	2068	2173	2078	2159	2170	2282	2165	2065	2177	2470(49)
	2269	2288	2304	2208	2188	2182		2280	2215	2236	
	2276	2295	2332	2263	2245			2345	2265	2287	
	2429	2449									
	2671	2703									
	2681	2713									
	2852	2885									
	3122	3166									
	3134	3178									
$5/2^+$	2250	2267	2401	2224	2303	2178		2280	2225	2253	
	2257	2275		2260	2252	2210		2345	2265	2312	
	2646	2676									
	2656	2686									
	3092	3135									
	3102	3146									
$7/2^+$	2233	2249	2369	2205	2321	2183		2295	2210	2292	
	2623	2652									
	3065	3107									

Table 8. Comparison of present masses with other approaches based on J^P value with negative parity described in the increasing order for all possible spin-parity assignment (in MeV).

J^P	$Mass_{\rm cal}1$	$Mass_{\rm cal}2$	[47]	[48]	[49]	[7]	[50]	[51]	[52]	[54]	[55]
$1/2^-$	1987	1996	1941	2015	1957	1923	1837	1950	2020	1992	1944(56)
	1983	2001	2463					2410		2456	2716(118)
	2345	2363	2580					2490		2498	
	2352	2370							2550		
	2758	2788									
	2767	2797									
	3218	3264									
	3229	3276									
$3/2^-$	1978	1985	2038	2015	2012	1953	1978	2000	2020	1976	2049(32)
	1983	1991	2537				2604	2440		2446	2755(67)
	2332	2349	2636					2495		2507	
	2339	2356								2524	
	2622	2653								2564	
	2740	2770								2594	
	2749	2779									
	3072	3115									
	3196	3240									
	3207	3252									
$5/2^-$	1970	1997	2653					2490		2528	
	2321	2338								2534	
	2585	2614								2554	
	2595	2625								2617	
	2726	2755									
	3027	3069									
	3039	3081									
	3178	3221									
$7/2^-$	2562	2590	2599							2531	
	2998	3040								2577	
$9/2^-$	2521	2548	2649							2606	
	2949	2999									

The present model has attempted to distinguish all the possible spin-parity assignment of the excited states. However, due to limited data obtained by various compared models, exact state-wise comparison is not possible. Thus, this study is expected to provide a possible range of masses for upcoming experiments, which shall identify the existence of a particular state.

IV. REGGE TRAJECTORIES

Regge trajectories have been of importance in spectroscopic studies. The total angular momentum J and principal quantum number n are plotted against the square of resonance mass M^2 to obtain the non-intersecting and linearly fitted lines. Figure 2 shows a linear behaviour with almost all the points following the trend for $n-M^2$. Figures 3 and 4 are plotted with few natural and unnatural parity states for available results. These plots point toward the fact that the spin-parity assignment of a given state in the present calculation could possibly be correct. The linear fitting parameters are mentioned in the respective plots.

$$J = aM^2 + a_0, \tag{17a}$$

$$n = bM^2 + b_0. \tag{17b}$$

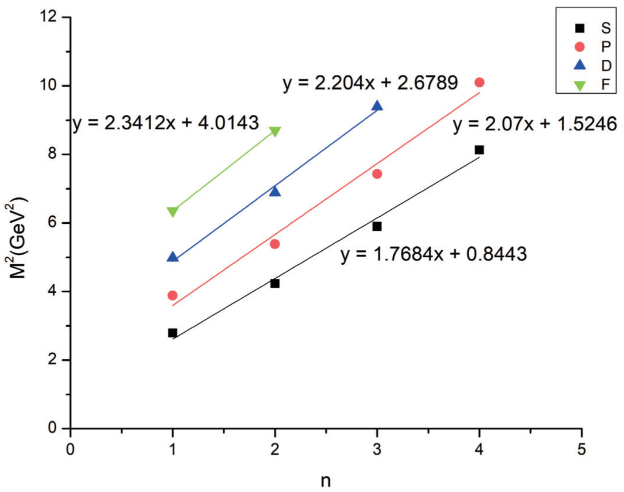

Fig. 2. (color online) Regge trajectory $n \rightarrow M^2$ for S, P, D and F state masses and linearly fitted.

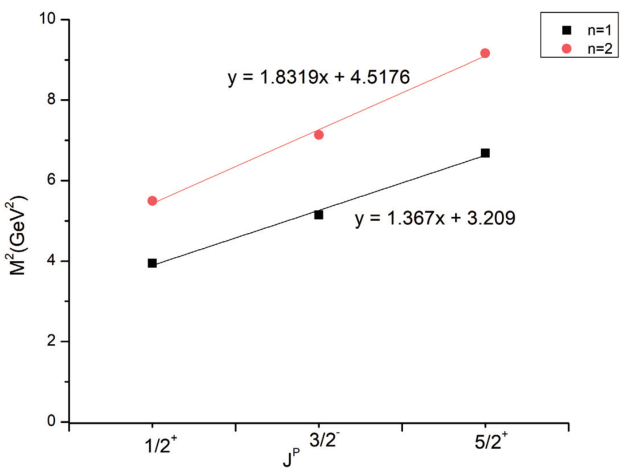

Fig. 3. (color online) Regge trajectory $J \rightarrow M^2$ for natural parity.

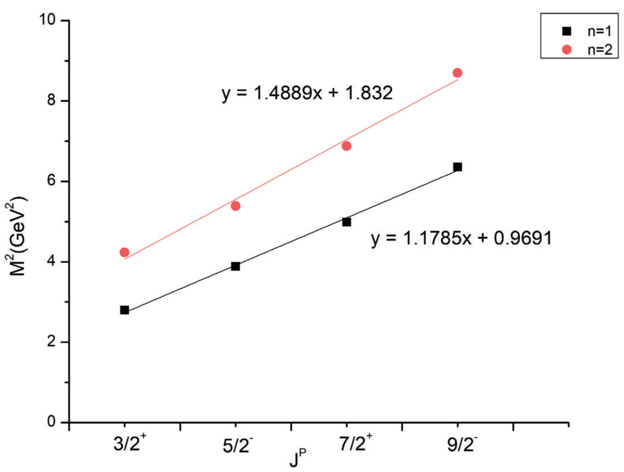

Fig. 4. (color online) Regge trajectory $J \rightarrow M^2$ for unnatural parity.

Faustov *et al.* [47] have shown the total angular momentum J against the square of mass trajectories using the resonances obtained with the relativistic quark model. The slope and intercept values have been given as 0.712 ± 0.002 and –0.504 ± 0.007, respectively. The similar plot with natural parity for present masses gives the value as 1.1785 ± 0.048 and 0.9691 ± 0.154 respectively. The values for slope and intercept with standard error for (n,M^2) plot are listed in Table 9. However, due to the lack of more experimental data, we are unable to comment on the exact comparison of the values.

Table 9. Regge slopes and intercepts for (n,M^2).

Trajectory	b	b_0
S	1.76838 ± 0.12837	0.84426 ± 0.35156
P	2.07004 ± 0.1839	1.52458 ± 0.50364
D	2.20397 ± 0.17905	2.67895 ± 0.3868
F	2.34116	4.01428

V. MAGNETIC MOMENT

The electromagnetic properties of baryons are a challenging realm, especially for short-lived $J^P = 3/2^+$ decuplet baryons. Many theoretical approaches have attempted to investigate strange baryon magnetic moments. They served as an asset for the study of decay properties as well as intrinsic dynamics of quarks. The generalized form of magnetic moment is [45]

$$\mu_B = \sum_q \langle \phi_{sf} | \mu_{qz} | \phi_{sf} \rangle, \tag{18}$$

where ϕ_{sf} is the spin-flavour wave function. The contribution from individual quark appears as

$$\mu_{qz} = \frac{e_q}{2m_q^{\text{eff}}} \sigma_{qz}, \tag{19}$$

e_q being the quark charge, σ_{qz} being the spin orientation, and m_q^{eff} being the effective mass, which may vary from the model based quark mass due to interactions. In case of Ω, $\sigma_{qz} = s\uparrow s\uparrow s\uparrow$, which leads to $3\mu_s$. Table 10 summarizes the calculated magnetic moment alongwith other comparison results [56–62].

VI. CONCLUSION

A hypercentral Constituent Quark Model with a linear confining term, spin-dependent terms, and a correction term has been helpful to obtain mass spectra for higher excited states up to nearly 3 GeV. Even though the scarcity of experimental data does not allow us to completely validate the findings, comparisons with theoretic-

Table 10. Comparison of ground state magnetic moment (in μ_N).

Present	Exp	[56]	[56]	[57]	[58]	[59]	[60]	[61]	[62]
−1.68	−2.02	−1.67	−1.90	−2.06	−1.95	−1.61	−2.08	−2.01	−1.84

al models with varied assumptions are of keen interest. The low-lying states are in good accordance with some models but not exactly matching for higher J^P values.

It is noteworthy that the current findings could not comment on the debated state of Ω(2012) for molecular structure. However, the mass varies within 30 MeV with $J^P = 3/2^-$, which may be identified as a negative parity state of 1P family. As the J^P value for any other state is not experimentally known, an exact comparison still depends on the availability of more findings in future. However, the probable spin-parity assignment according to the obtained value can be given. Thus, Ω(2250) could be the $1D_{5/2^+}$ state, Ω(2380) could be $2P_{1/2^-}$, and Ω(2470) may be associated to 3S with $3/2^+$.

The magnetic moment differs by $0.5\mu_n$ from PDG and other results. The Regge trajectories show a linear nature, hinting that the spin-parity assignments may be correct. However, the validation of any of the results depends on the future experimental facilities to exclusively study the strange baryon properties, especially by $\bar{P}$ANDA at FAIR-GSI [10] and BESIII [11].

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